The Abelian Bohm vacuum is defined by

\[ B = \nabla \times W = \nabla \times A + \omega \times A \]  

(1)

where

\[ W = W(0, \omega), \quad A = A(0, \nu) \]  

(2)

Here \( \omega \) is the spin current and \( \nu \) is the drift.

Therefore the geometry of the AB vacuum is defined by

\[ \nabla \times \omega = 0 \]  

(3)

and

\[ \nabla \times \nu = 2 \nu \times \omega \]  

(4)

Now use:

\[ \nabla \cdot (\nabla \times \nu) = 0 \]  

(5)

From eqs (4) and (5):

\[ \nabla \cdot (\nabla \times \omega) = 0 \]  

(6)

\[ = \omega \cdot \nabla \times \nu - \nu \cdot \nabla \times \omega \]

For eqs (3) and (6):

\[ \omega \cdot \nabla \times \nu = 0 \]  

(7)

i.e.

\[ W \cdot \nabla \times A = 0 \]  

(8)

Under condition (8), the potentials \( W \) and \( A \)
2) are non-zero, but the magnetic flux density \( \mathbf{B} \) is zero.

In the well-known Cherenkov experiment, an electron beam is deflected in regions where there is no magnetic field. This effect is described by:

\[
\mathbf{p} \rightarrow \mathbf{p} - eA_{\text{vac}}
\]

where \( A_{\text{vac}} \) is the AB vacuum potential. It is concluded that an ESR experiment could be carried out in regions where there is no magnetic field.

The electric field strength in ECE theory is defined by:

\[
\mathbf{E} = - \nabla \phi_{\text{w}} - \frac{\partial \mathbf{W}}{\partial t}
\]

so the electric AB effect occurs when:

\[
\nabla \phi_{\text{w}} = - \frac{\partial \mathbf{W}}{\partial t}
\]

In terms of curvatures:

\[
\mathbf{B} = \mathbf{W}^{(6)} R_{\text{spin}} - 12
\]

\[
\mathbf{E} = c \mathbf{W}^{(6)} R_{\text{ads}} - 13
\]

where \( R_{\text{spin}} \) and \( R_{\text{ads}} \) are the spin and orbital curvature vectors.

Therefore the AB effect occurs in regions where
3) The tetrad and spin connection are finite but vanishing. There is no spin or orbital current. Similarly the torsion tensor is zero in the Abelian case, but the torsion tensor is non-zero.

In minimal notation:

\[ T = d \wedge \Psi + c \wedge \Psi = 0 \quad (14) \]
\[ R = d \wedge \omega + c \wedge \omega = 0 \quad (15) \]

So

\[ d \wedge \Psi = -c \wedge \Psi \quad (16) \]

and

\[ d \wedge \omega = -c \wedge \omega \quad (17) \]

with

\[ T = R = 0 \quad (18) \]

Vacuum Defined by Absence of Charge Current Density

Consider the ECEF electromagnetic field equations:

\[ \nabla \cdot B = 0 \quad (19) \]
\[ \nabla \cdot E = \sigma_0 \quad (20) \]
\[ \frac{dB}{dt} + \nabla \times E = -\left( \sigma_0 \frac{cB}{\varepsilon_0} + \nabla \times E \right) \quad (21) \]
\[ \nabla \times B - \frac{1}{c^2} \frac{dE}{dt} = \frac{\sigma_0 E + \nabla \times B}{\varepsilon_0} \quad (22) \]

where

\[ \sigma_0 = 2 \left( \frac{\alpha}{\varepsilon_0} - c \right) \quad (23) \]
\[ \kappa = 2 \left( \frac{\alpha}{\varepsilon_0} - c \right) \quad (24) \]
In the absence of charge current density:

\[ \nabla \cdot \mathbf{B} = 0 \quad - (25) \]
\[ \nabla \cdot \mathbf{E} = 0 \quad - (26) \]
\[ \frac{dB}{dt} + \nabla \times \mathbf{E} = 0 \quad - (27) \]
\[ \frac{\partial}{\partial t} \mathbf{B} - \frac{1}{c^2} \frac{d\mathbf{E}}{dt} = 0 \quad - (28) \]

and these are no magnetic or electric charge current densities, but the electromagnetic field is not zero.

In the Aether, the vacuum has no charge densities and the fields are zero, but their potentials are not zero.

One possible solution for Eqs. (25) to (28) is

\[ \Phi_0 = 0 \quad - (29) \]
\[ \Psi = 0 \quad - (30) \]

so

\[ \mathbf{V}_0 = \mathbf{r} \psi_0 \quad - (31) \]
\[ \mathbf{V} = \mathbf{r} \psi \quad - (32) \]

From Eq. (32) it follows that

\[ \mathbf{\omega} \times \mathbf{A} = \mathbf{A} \quad \mathbf{\omega} \times \mathbf{\omega} \]

so

\[ \mathbf{\omega} \times \mathbf{\omega} \]

or in the absence of charge current density, \( \mathbf{0} \).
In the presence of electromagnetic fields:

\[ b = \nabla \times W = \nabla \times A \neq 0 \quad -(34) \]

It is seen that eq. (6) of the AB vacuum is satisfied by eq. (32), because the latter implies:

\[ \nabla \cdot (\nabla \times A) = 0 \quad -(35) \]

Eq. (4) of the AB vacuum is also satisfied by eq. (32) provided that:

\[ \nabla \times A = 0 \quad -(36) \]

but eqs. (32) and (35) imply:

\[ \nabla \times A = 0 \quad -(37) \]

so it is concluded that eq. (35) implies:

\[ b = 0 \quad -(38) \]

Therefore it is found that charge current densely vanish and fields remain finite solutions other than eq. (32) must be found, and \( \nabla \cdot \mathbf{E} \) and \( \nabla \times \mathbf{B} \) must remain finite. The equations to be solved are:

\[ \nabla \cdot \mathbf{B} = 0 \quad -(39) \]

\[ \nabla \cdot \mathbf{E} = 0 \quad -(40) \]

\[ \nabla \times \mathbf{B} + \kappa \mathbf{E} = 0 \quad -(41) \]

\[ \kappa \mathbf{E} + \nabla \times \mathbf{B} = 0 \quad -(42) \]
From Eqs. (41) and (43) :
\[ \mathbf{b} = -\frac{1}{\kappa_0 c} \kappa \times \mathbf{E} \quad (44) \]
and
\[ \mathbf{E} = -\frac{c}{\kappa_0} \kappa \times \mathbf{b} \quad (45) \]
Therefore by Eqs. (39) and (40):
\[ \kappa \cdot (\kappa \times \mathbf{b}) = 0 \quad (46) \]
and
\[ \kappa \cdot (\kappa \times \mathbf{E}) = 0 \quad (47) \]
Now use:
\[ \kappa \cdot (\kappa \times \mathbf{A}) = \mathbf{b} \cdot (\kappa \times \kappa) = 0 \quad (48) \]
and
\[ \kappa \cdot (\kappa \times \mathbf{E}) = \mathbf{E} \cdot (\kappa \times \kappa) = 0 \quad (49) \]
So the solutions to Eqs. (44) and (45) are:
\[ \frac{\partial \mathbf{b}}{\partial t} + \nabla \times \mathbf{E} = 0 \quad (50) \]
and
\[ \nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = 0 \quad (51) \]
The plane wave solution of Eqs. (50) and (51) is:
7) From eq. (45):
\[ \mathbf{k} \times \mathbf{E} = -\frac{c}{\kappa_0} \mathbf{k} \times (\mathbf{k} \times \mathbf{B}) - (52) \]

so
\[ \kappa_0^2 \mathbf{B} = \mathbf{k} \left( \mathbf{k} \cdot \mathbf{B} \right) - \kappa^2 \mathbf{B} - (53) \]

where
\[ \kappa^2 = \mathbf{k} \cdot \mathbf{k} - (54) \]
\[ (\kappa^2 + \kappa_0^2) \mathbf{B} = \mathbf{k} \left( \mathbf{k} \cdot \mathbf{B} \right) - (55) \]

i.e.

The plane wave solutions of eqs. (50) and (51) are:
\[ \mathbf{b} = b(0) (i \mathbf{i} + j \mathbf{j}) e^{i \varphi} - (56) \]

and
\[ \mathbf{E} = E(0) (i - j \mathbf{j}) e^{i \varphi} - (57) \]

From eqs. (55) and (56):
\[ (\kappa^2 + \kappa_0^2) (i \mathbf{i} + j \mathbf{j}) = \mathbf{k} \left( \mathbf{k} \cdot (i \mathbf{i} + j \mathbf{j}) \right) - (58) \]

Comparing real parts:
\[ (\kappa^2 + \kappa_0^2) j = \kappa_y \kappa - (59) \]

Comparing imaginary parts:
\[ (\kappa^2 + \kappa_0^2) i = \kappa_x \kappa - (60) \]

Adding:
\[ (\kappa^2 + \kappa_0^2) (i + j) = (\kappa_x + \kappa_y) \kappa - (61) \]

so
\[ \kappa = \left( \frac{\kappa^2 + \kappa_0^2}{\kappa_x + \kappa_y} \right) (i + j) - (62) \]