Consider the rigorous energy equation of a free electron:

\[ E^2 = c^2 p^2 + m^2 c^4 \quad - (1) \]

where \( E \) is the total energy:

\[ E = \gamma mc^2 = \frac{p}{c} \quad - (2) \]

and \( p \) the relativistic momentum:

\[ p = \gamma p_0 = \frac{m \dot{c}}{c} \quad - (3) \]

In this case there is no potential energy, so the relativistic Hamiltonian is:

\[ H = E. \quad - (4) \]

Eqn. (1) is:

\[ E^2 - m^2 c^4 = (E - mc^2)(E + mc^2) = c^2 p^2 \quad - (5) \]

so

\[ E = \frac{c^2 p^2}{E + mc^2} + mc^2 \quad - (6) \]

\[ = \frac{\gamma^2}{1 + \gamma} \frac{p_0^2}{m} + mc^2 \]

From eqn. (2)

\[ \gamma = \frac{E_0}{mc^2} \quad - (7) \]

where \( \omega_0 \) is the angular frequency of the electron.
matter wave, and $n$ is the electron mass. Therefore:

\[ x = \frac{y^2}{1 + \beta} = \left( \frac{\ell c}{m c^2} \right)^2 \left( 1 + \frac{\ell c}{m c^2} \right)^{-1} \]  

Eq. (8) is true for any particle of mass $m$, including the photon.

Now apply a magnetic flux density $B \to \mathbf{A}$

electron beam. The resulting Hamiltonian is:

\[ E = H = \frac{x}{m} \left( \frac{p_0 - e A}{m} \right) \cdot \left( \frac{p_0 - e A}{m} \right) + mc^2 \]  

This is quantized as follows:

\[ H \psi = \frac{2x}{m} \left( -i \hbar \mathbf{\nabla} - e \mathbf{A} \right) \cdot \left( \frac{p_0 - e A}{m} \right) + mc^2 \psi \]  

In the $su(2)$ basis:

\[ H \psi = \frac{x}{m} \sigma \cdot \left( -i \hbar \mathbf{\nabla} - e \mathbf{A} \right) \sigma \cdot \left( \frac{p_0 - e A}{m} \right) \psi + mc^2 \psi \]  

giving the interaction Hamiltonian:

\[ H_{\text{int}} \psi = -x e \hbar \frac{\sigma \cdot \mathbf{\nabla} \times \mathbf{A}}{m} \psi \]
3) The spin angular momentum of the electron is:

\[ S = \frac{\hbar}{2} \sigma \]  \hspace{1cm} -(13) \]

so:

\[ H \cdot \sigma = -2xe \frac{\sigma \cdot \mathbf{B}}{m} \]  \hspace{1cm} -(14) \]

where

\[ x = \left( \frac{\hbar \omega}{m} \right)^{\frac{3}{2}} \left( 1 + \frac{\hbar \omega}{m} \right)^{-1} \rightarrow \frac{1}{\hbar \omega}, \quad \frac{1}{2} \]  \hspace{1cm} -(15) \]

The usual theory assumes:

\[ \hbar \omega = mc^2 \]  \hspace{1cm} -(16) \]

In eqn. (14):

\[ S_z \sigma_z = m_s \hbar \sigma_z \]  \hspace{1cm} -(17) \]

For a magnetic field aligned with the Z axis:

\[ E = -2xe \frac{m_s B_z}{m} \]  \hspace{1cm} -(18) \]

where

\[ m_s = \frac{1}{2} \text{ and } -\frac{1}{2} \]  \hspace{1cm} -(19) \]

The ESR resonance frequency is:

\[ \omega_{ESR} = 2xe \frac{B_z}{m} \]  \hspace{1cm} -(20) \]
\[ \omega_{\text{ESR}} = 2 \left( \frac{\hbar \omega}{mc^2} \right)^2 \left( 1 + \frac{\hbar \omega}{mc^2} \right)^{-1} \frac{e}{m} B z \] - (21)

This result is directly testable in a relativistic electron beam.

In the presence of a vacuum potential \( \mathbf{A}_{\text{vac}} \), there is an additional magnetic flux density:

\[ \mathbf{B}_{\text{vac}} = \mathbf{v} \times \mathbf{A}_{\text{vac}} \] - (22)

and the next rate will apply to theory of QFT-31P.

to this situation.