Orbit Equations in m Space, Developed Version

Consider the infinitesimal line element of m space:

$$ds^2 = c^2 d\tau^2 = c^2 m(r) dt^2 - \frac{dr^2}{m(r)} - r^2 d\phi^2 \quad -(1)$$

and define the linear velocity as:

$$v^2 dt^2 = \frac{dr^2}{m(r)} + r^2 d\phi^2 \quad -(2)$$

Since $m(r)$ is considered to be independent of time:

$$v^2 = \frac{1}{m(r)} \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\phi}{dt}\right)^2 \quad -(3)$$

$$= \frac{1}{m(r)} \dot{r}^2 + r^2 \dot{\phi}^2$$

It follows that:

$$c^2 d\tau^2 = c^2 m(r) dt^2 - v^2 dt^2 \quad -(4)$$

So

$$\gamma = \frac{dt}{d\tau} = \left(m(r) - \frac{v^2}{c^2}\right)^{-1/2} \quad -(5)$$

is the Lorentz factor in m space.

The structure of the field equations of ECE2 theory is the structure of the field equations in Minkowski spacetime. However ECE2 is generally covariant and developed in a space with finite torsion and curvature. It is therefore considered that the linear velocity in m theory is:

$$\underline{v} = \gamma \underline{\dot{r}} \quad -(6)$$

From eq. (3):

$$v^2 = \underline{\dot{r}} \cdot \underline{\dot{r}} = \frac{1}{m(r)} \dot{r}^2 + r^2 \dot{\phi}^2$$

$$-(7)$$

So
$$\underline{\dot{r}} = \frac{\dot{r}}{n(r)^{1/2}}\,\underline{e}_r + r\dot{\phi}\,\underline{e}_\phi \quad -(8)$$
where $\underline{e}_r$ and $\underline{e}_\phi$ are the unit vectors of the plane polar system.

It follows that:
$$\underline{\ddot{r}} = \frac{1}{n(r)^{1/2}}\frac{d}{dt}\left(\dot{r}\,\underline{e}_r\right) + \frac{d}{dt}\left(r\dot{\phi}\,\underline{e}_\phi\right)$$
$$= \frac{1}{n(r)^{1/2}}\left(\ddot{r}\,\underline{e}_r + \dot{r}\,\dot{\underline{e}}_r\right) + \dot{r}\dot{\phi}\,\underline{e}_\phi + r\ddot{\phi}\,\underline{e}_\phi + r\dot{\phi}\,\dot{\underline{e}}_\phi$$
$$= \frac{1}{n(r)^{1/2}}\left(\ddot{r}\,\underline{e}_r + \dot{r}\dot{\phi}\,\underline{e}_\phi\right) + \left(\dot{r}\dot{\phi} + r\ddot{\phi}\right)\underline{e}_\phi - r\dot{\phi}^2\,\underline{e}_r$$
$$= \left(\frac{\ddot{r}}{n(r)^{1/2}} - r\dot{\phi}^2\right)\underline{e}_r + \left(r\ddot{\phi} + \dot{r}\dot{\phi}\left(1 + \frac{1}{n(r)^{1/2}}\right)\right)\underline{e}_\phi \quad -(9)$$

The acceleration due to gravity is:
$$\underline{g} = \frac{d}{dt}\left(\gamma\,\underline{\dot{r}}\right)$$
$$= \frac{d\gamma}{dt}\,\underline{\dot{r}} + \gamma\,\underline{\ddot{r}} \quad -(10)$$

So in ECE2 theory:
$$\underline{g} = \frac{d\gamma}{dt}\,\underline{\dot{r}} + \gamma\,\underline{\ddot{r}} = -\underline{\nabla}\,\underline{\Phi} + \underline{\Omega}\,\underline{\Phi} \quad -(11)$$
[wh]e[re]
$$\underline{\Phi} = -\frac{MG}{r} \quad -(12)$$
the gravitational potential.

It follows that:

$$\underline{g} = \frac{d\gamma}{dt}\left(\frac{\dot{r}}{m(r)^{1/2}}\,\underline{e}_r + r\dot{\phi}\,\underline{e}_\phi\right)$$

$$+ \gamma\left(\left(\frac{\ddot{r}}{m(r)^{1/2}} - r\dot{\phi}^2\right)\underline{e}_r + \left(r\ddot{\phi} + \dot{r}\dot{\phi}\left(1 + \frac{1}{m(r)^{1/2}}\right)\right)\underline{e}_\phi\right)$$

$$= \left(-\frac{mG}{r^2} + \Omega_r \Phi\right)\underline{e}_r \qquad -(13)$$

$$= -\frac{mG}{r}\left(\frac{1}{r} + \Omega_r\right)\underline{e}_r$$

Here it has been assumed that:

$$\underline{\Omega} = \Omega_r\,\underline{e}_r + \Omega_\phi\,\underline{e}_\phi \qquad -(14)$$

and that

$$\Omega_\phi = 0 \qquad -(15)$$

From eq. (13) - (15):

$$\frac{d\gamma}{dt}\frac{\dot{r}}{m(r)^{1/2}} + \gamma\left(\frac{\ddot{r}}{m(r)^{1/2}} - r\dot{\phi}^2\right) = -\frac{mG}{r}\left(\frac{1}{r} + \Omega_r\right) \qquad -(16)$$

is the generally covariant Leibniz equation in m space.

and

$$\frac{dL}{dt} = r\dot{\phi}\frac{d\gamma}{dt} + \gamma\left(r\ddot{\phi} + \dot{r}\dot{\phi}\left(1 + \frac{1}{m(r)^{1/2}}\right)\right) = 0 \qquad -(17)$$

is the equation of conservation of angular momentum L. The orbit is given by solving eqs. (16) and (17) simultaneously, and L is found by integrating eq. (17) numerically.

i) As in UFT190, the orbit can also be found by numerical integration of:

$$\frac{dr}{d\phi} = r^2 \left(\frac{1}{b^2} - m(r) \left(\frac{1}{a^2} + \frac{1}{r^2} \right) \right)^{1/2}, \quad -(18)$$

and in ECE cosmology:

$$m(r) = 2 - \exp\left(2 \exp\left(-\frac{r}{R} \right) \right) \quad -(19)$$

where R is a characteristic distance in the universe.

In papers such as UFT108 and UFT190, it was shown that orbital shrinkage can be explained with m theory.

The Lagrangian of the above theory is found from:

$$\underline{p} = m\underline{v} = m\gamma \underline{\dot{r}} = \frac{\partial \mathcal{L}}{\partial \underline{\dot{r}}} \quad -(20)$$

where:

$$\gamma = \frac{dt}{d\tau} = \left(m(r) - \frac{v^2}{c^2} \right)^{-1/2} \quad -(21)$$

$$= \left(m(r) - \frac{\underline{\dot{r}} \cdot \underline{\dot{r}}}{c^2} \right)^{-1/2}$$

This guarantees that the Lagrangian and relativistic theories give the same orbit equations (16) and (17).