414(7): The Spin Connection due to ECE2 Covariance

In the previous note the orbital equations were defined as:

$$F = m\gamma^3 \dot{v} = -\frac{mMG}{r^2} \quad -(1)$$

and

$$\frac{dL}{dt} = \frac{d}{dt}\left(\gamma m r^2 \dot{\phi}\right) = 0 \quad -(2)$$

However, there must be a spin connection due to ECE2 covariance, so the fully rigorous eqs. (1) and (2) are

$$F = m\gamma^3 \dot{v} = -\frac{mMG}{r^2} + \Omega_r \Phi m \quad -(3)$$

and

$$\frac{dL}{dt} = \frac{d}{dt}\left(\gamma m r^2 \dot{\phi}\right) = 0 \quad -(4)$$

Here

$$\Phi = -\frac{MG}{r} \quad -(5)$$

is the gravitational potential.

Therefore relativity itself produces a spin connection, a vacuum force and orbital precession.

The Newtonian counterpart of Eq. (3) is:

$$F_N = m\dot{v} = -\frac{mMG}{r^2} \quad -(6)$$

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From eqs (1) and (6):

$$-\frac{MG}{r^2}\left(\gamma^3 - 1\right) = \Omega_r \Phi = -\Omega_r \frac{MG}{r}$$

so

$$\Omega_r = \frac{1}{r}\left(\left(1 - \frac{v^2}{c^2}\right)^{-3/2} - 1\right) \quad -(8)$$

Therefore eqs. (2) and (3) must be solved simultaneously.

$$F = m\gamma^3 \dot{v} = -\frac{mMG}{r^2} + \frac{1}{r} m \Omega_r \Phi \quad -(9)$$

$$\frac{d}{dt}\left(\gamma m r^2 \dot{\phi}\right) = 0 \quad -(10)$$

where:

$$\Omega_r = \frac{1}{r}\left(\left(1 - \frac{v^2}{c^2}\right)^{-3/2} - 1\right) \quad -(11)$$

and

$$\Phi = -\frac{MG}{r} \quad -(12)$$

Therefore:

$$F = m\gamma^3 \dot{v} = -\frac{mMG}{r^2}\left(1 + \gamma^3 - 1\right) \quad -(13)$$

i.e

$$\boxed{F = \gamma^3 m \dot{v} = -\gamma^3 \frac{mMG}{r^2} \quad -(14)}$$

and

$$\boxed{\frac{d}{dt}\left(\gamma m r^2 \dot{\phi}\right) = 0 \quad -(15)}$$

Therefore the simultaneous equations to be solved in this case are:

$$\dot{v} = \ddot{r} - r\dot{\phi}^2 = -\frac{MG}{r^2} \quad -(16)$$

and

$$r\dot{\phi}\frac{d\gamma}{dt} + \gamma\left(2\dot{r}\dot{\phi} + r\ddot{\phi}\right) = 0 \quad -(17)$$

In the next note the effect of:

$$\phi' = \phi + \omega_1 t \quad -(18)$$

on eqs. (16) and (17) will be investigated.