410(4): Origin and Derivation of the Precession due to ECE2 Covariant General Relativity

Consider the invariance of phase under four rotation:

$$\phi = \kappa^\mu x_\mu = \kappa^{\mu\prime} x_\mu^\prime \quad -(1)$$

$$= \omega t - \underline{\kappa} \cdot \underline{r} = \omega' t' - \underline{\kappa}' \cdot \underline{r}'$$

Here:

$$\kappa^\mu = \left(\frac{\omega}{c}, \underline{\kappa} \right), \quad x^\mu = (ct, \underline{r}) \quad -(2)$$

In its infinitesimal format:

$$\phi = \omega dt - \underline{\kappa} \cdot d\underline{r} = \omega' dt' - \underline{\kappa}' \cdot d\underline{r}' \quad -(3)$$

A particle in the moving frame κ' does not move with respect to κ' so:

$$dr' = 0 \quad -(4)$$

and

$$\phi = \omega dt - \underline{\kappa} \cdot d\underline{r} = \omega' dt' \quad -(5)$$

In the notation of previous notes:

$$\phi = \omega dt - \underline{\kappa} \cdot d\underline{r} = \omega_0 dt_1 = \omega_0 d\tau \quad -(6)$$

where ω_0 is the rest frequency and τ the proper time.

Now use the de Broglie / Einstein equations:

$$E_n = \hbar\omega = \gamma m c^2 \quad -(7)$$

$$p = \hbar \underline{\kappa} = \gamma m \underline{v} \quad -(8)$$

to find that the rest angular frequency is:

$$\omega_0 = \frac{mc^2}{\hbar} \quad -(9)$$

2) This is the angular frequency in frame K'. In frame K, the laboratory or observer frame:

$$\omega = \gamma \omega_0 \quad -(10)$$

In the preceding notes it has been shown that:

$$dt = \gamma \, dt_1 \quad -(11)$$

It follows that the phase change between frame K and frame K' is:

$$\Delta \phi = \omega t - \omega_0 t_1 = \underline{K} \cdot \underline{r}$$
$$= \omega t - \frac{\omega t}{\gamma^2} = \omega t \left(1 - \frac{1}{\gamma^2}\right) \quad -(12)$$

For one revolution: $\omega t = 2\pi \quad -(13)$

so $$\Delta \phi = 2\pi \left(1 - \frac{1}{\gamma^2}\right) = \underline{K} \cdot \underline{r} \quad -(14)$$
$$= 2\pi \left(\frac{\gamma^2 - 1}{\gamma^2}\right) = 2\pi (\gamma - 1) \frac{(\gamma + 1)}{\gamma^2}$$

The definition of precession used in the "usual" Thomas precession is:

$$\Delta \phi = 2\pi (\gamma - 1) = \frac{\gamma^2}{\gamma + 1} \underline{K} \cdot \underline{r} \quad -(15)$$

The correct expression is:

$$\boxed{\Delta \phi = \omega t - \omega_0 t_1 = 2\pi \left(1 - \frac{1}{\gamma^2}\right) = \underline{K} \cdot \underline{r}} \quad -(16)$$

In the low velocity limit eq. (16) reduces to:

$$\Delta \phi = 2\pi \left(1 - \frac{1}{\gamma^2} \right) = 2\pi \frac{v^2}{c^2} \quad -(17)$$

and this is true for all γ and all v. Eq. (17) is the net result of fundamental theory, and replaces the existing "Wikipedia" result:

$$\Delta \phi =? \; 2\pi (\gamma - 1) \quad -(18)$$

Eq. (17) is an exact result. If de Sitter present:

$$v^2 = v_N^2 + 3\omega^2 r^2 \quad -(19)$$

and the universal law of precession is:

$$\boxed{\Delta \phi = \omega t - \omega_0 t_1 = 2\pi \frac{v^2}{c^2} = \frac{2\pi}{c^2} \left(v_N^2 + 3\omega^2 r^2 \right) \quad -(20)}$$

If de Sitter rotation is absent:

$$\Delta \phi = \omega t - \omega_0 \tau = 2\pi \frac{v_N^2}{c^2} \quad -(21)$$

and this is precession due to time dilatation and length contraction, in which

$$\gamma = \frac{\ell'}{\ell} = \frac{\Delta t}{\Delta \tau} \quad -(22)$$

$$\Delta \phi = 2\pi \left(1 - \frac{\ell^2}{\ell'^2} \right) = 2\pi \left(1 - \left(\frac{\Delta \tau}{\Delta t} \right)^2 \right) \quad -(23)$$
