409(7): The Conventional and New Methods of Calculating Thomas Precession, i.e. ECE2 Precession.

The conventional method was described in UFT 110, following a wikipedia source uncritically. This source adapted the 1916 de Sitter method to the lower Thomas precession. The conventional method uses the infinitesimal line element:

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi'^2 \quad -(1)$$

generated by the de Sitter rotation:

$$\phi' = \phi + \omega t \quad -(2)$$

and

$$v_\theta = \omega r \quad -(3)$$

Eqns. (1) and (2) give:

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - 2\omega r^2 d\phi\, dt - \omega^2 r^2 dt^2 \quad -(4)$$

$$= (c^2 - v_\theta^2)\, dt^2 - 2\omega r^2 d\phi\, dt - dr^2 - r^2 d\phi^2 \quad -(5)$$

Now use:

$$\omega = \frac{d\phi}{dt} \quad -(6)$$

so

$$d\phi = \omega\, dt \quad -(7)$$

and it follows that:

$$2\omega r^2 d\phi\, dt = 2\omega^2 r^2 dt^2 = 2 v_\theta^2 dt^2 \quad -(8)$$

So:

$$ds^2 = (c^2 - 3 v_\theta^2)\, dt^2 - (dr^2 + r^2 d\phi^2) \quad -(9)$$

Finally use the definition of the Newtonian velocity:

$$v_N^2 = \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\phi}{dt}\right)^2 \quad -(10)$$

to find that

$$dr^2 + r^2 d\phi^2 = v_N^2 dt^2 \quad -(11)$$

Therefore eq. (9) becomes:

$$ds^2 = (c^2 - v_N^2 - 3v_\theta^2)\,dt^2 \quad -(12)$$

i.e.

$$ds^2 = c^2 d\tau^2 = \left(1 - \frac{v^2}{c^2}\right) c^2 dt^2 \quad -(13)$$

here

$$v^2 = v_N^2 + 3\omega^2 r^2 = v_N^2 + 3v_\theta^2 \quad -(14)$$

without the de Sitter rotation (2) Eq (13) is:

$$ds^2 = c^2 d\tau^2 = \left(1 - \frac{v_N^2}{c^2}\right) c^2 dt^2 \quad -(15)$$

$$= c^2 dt^2 - dr^2 - r^2 d\phi^2 \quad -(16)$$

In eq. (15), the infinitesimal of time has been changed to

$$dt' = \left(1 - \frac{v_N^2}{c^2}\right)^{1/2} dt = \frac{dt}{\gamma} \quad -(17)$$

here the Lorentz factor is:

$$\gamma = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} \quad -(18)$$

So

$$dt = \gamma\, dt' \quad -(19)$$

a result of invariance under four rotation:

$$x^\mu x_\mu = x'^\mu x'_\mu. \quad -(20)$$

Eq. (19) produces the precession:

$$\Delta\phi = 2\pi(\gamma - 1) \quad -(21)$$

$$= \omega_0(dt - dt') = (\gamma - 1)\,\omega_0\, dt'$$

This means that the orbit of m around M is no longer closed. It advances by $\Delta\phi$ per 2π radians

The expression for $\Delta\phi$ is:

$$\Delta\phi = 2\pi\left(\left(1-\frac{v_N^2}{c^2}\right)^{-1/2}-1\right) \xrightarrow[v_N \ll c]{} \pi\frac{v_N^2}{c^2} \quad -(22)$$

When de Sitter rotation is considered:

$$\Delta\phi = 2\pi\left(\left(1-\frac{v^2}{c^2}\right)^{-1/2}-1\right) \xrightarrow[v \ll c]{} \pi\frac{v^2}{c^2} \quad -(23)$$

where

$$\boxed{v^2 = v_N^2 + 3\omega^2 r^2} \quad -(24)$$

Any precession can be described precisely by its de Sitter angular velocity ω. For example planetary precessions, binary pulsar precession and pendulum precession. It is known that eq. (22) does not give an accurate description of planetary or binary pulsar precession. However eq. (23) gives the Lorentz factor for rotation.

The Conventional Method

This method was used uncritically in UFT 110 from a Wikipedia source that used the construction:

$$ds^2 = (c^2-v_\theta^2)\,dt^2 - 2\omega r^2\,d\phi\,dt - dr^2 - r^2\,d\phi^2$$

$$= \left(1-\frac{v_\theta^2}{c^2}\right)\left(c^2dt^2 - 2\Omega r^2\,d\phi\,dt\right) - dr^2 - r^2\,d\phi^2 \quad -(25)$$

where

$$\Omega := \omega\left(1-\frac{v_\theta^2}{c^2}\right)^{-1} \quad -(26)$$

It is now clear that the construction (25) is arbitrary, and yet another error / obscurity of Wikipedia.

Having made the construction (26), the precession was defined arbitrarily:

$$\Delta\phi = \Omega\, d\tau - \omega dt \quad -(27)$$

$$= 2\pi\left(\left(1-\frac{v_\theta^2}{c^2}\right)^{-1/2} - 1\right) \xrightarrow[v_\theta \ll c]{} \pi\frac{v_\theta^2}{c^2}$$

The correct result is eq. (23). It is known that eq. (27) does not the give the observed planetary and binary pulsar precessions because

$$\Delta\phi \xrightarrow{?} 0 \quad -(28)$$

when $\omega \to 0$. The correct result is

$$v \to v_N \quad -(29)$$

when $\omega \to 0$.

Finally, it is realized now that eq. (27) mixes frames of reference, because the proper time τ is the time in the moving frame while t is the time in the observer frame. So the Wikipedia article made a series of blunders. These seemed to arise from a flawed attempt to adapt the de Sitter method of 1916 to the Minkowski metric.