409(2): Relation between the Thomas velocity and Newtonian velocity

Consider the ECE2 covariant metric:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 \quad -(1)$$

in plane polar coordinates (r, ϕ). The Thomas velocity v_T is defined by the rotation:

$$\phi' = \phi + \omega t \quad -(2)$$

where

$$\omega r = v_T \quad -(3)$$

This rotation leads to the Thomas precession:

$$\Delta\phi_T = 2\pi\beta \quad -(4)$$

where

$$\beta = \left(1 - \frac{v_T^2}{c^2}\right)^{-1/2} - 1 \quad -(5)$$

$$\xrightarrow[v_T \ll c]{} \beta_0 = \frac{1}{2}\frac{v_T^2}{c^2}$$

The Lorentz factor γ is defined by the unrotated metric (1):

$$ds^2 = c^2 d\tau^2 = (c^2 - v_N^2)\, dt^2 \quad -(6)$$

where v_N is the Newtonian velocity. So

$$\gamma = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} = \frac{dt}{d\tau} \quad -(7)$$

From eqs. (1) and (6) the Newtonian velocity is defined by

$$v_N^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\phi}{dt}\right)^2 \quad -(8)$$

in the <u>unrotated</u> metric.

Under the condition:

$$v_T = v_N \quad -(9)$$

it follows that:

2)

$$\beta = \gamma - 1 \quad -(10)$$

More generally:
$$\beta \neq \gamma - 1 \quad -(11)$$

<u>Hypothesis</u>

Planetary precession is defined by the Thomas velocity v_T.

The precession of any planet is therefore:
$$\Delta \phi_T = 2\pi \left(\left(1 - \frac{v_T^2}{c^2} \right)^{-1/2} - 1 \right) \quad -(12)$$

In the solar system:
$$\Delta \phi_T = 2\pi \left(\frac{1}{2} \frac{v_T^2}{c^2} \right) \quad -(13)$$

where
$$\beta_0 = \frac{1}{2} \frac{v_T^2}{c^2} \quad -(14)$$

is the Thomas half. Therefore the precession of any object in the solar system, or in general anywhere in the universe can be described by eq. (12) and its low velocity limit, eq. (14).

Note carefully that the Newtonian eq. (8) does not produce a precessing orbit. It produces the conic section
$$r = \frac{\alpha}{1 + \epsilon \cos \phi} \quad -(15)$$

where α is the half right latitude and ϵ the eccentricity.

As in UFT 406, Eq. (15) produces:
$$v_N^2 = mG \left(\frac{2}{r} - \frac{1}{a} \right) = \frac{mG}{R_0} (1 + \epsilon) \quad -(16)$$

where
$$a = \frac{\alpha}{1 - \epsilon^2} \quad -(17)$$

is the semi major axis and

3)

$$R_0 = \frac{\alpha}{1+\epsilon} \quad -(18)$$

is the perihelion or distance of closest approach.

The Thomas and Newtonian velocities are different concepts, the former is relativistic and the latter is non-relativistic. The former produces Thomas precession and the latter does not. The two velocities can be the same only in the non-relativistic limit:

$$\frac{v_T^2}{c^2} = \frac{v_N^2}{c^2} \rightarrow 0 \quad -(19)$$

The Thomas factor β and Thomas half β_0 must always be defined by v_T, and not by v_N.

In the limit (19) for a Newtonian circular orbit,

$$v_T^2 = \omega^2 r^2 = v_N^2 = \frac{MG}{r} \quad -(20)$$

so

$$\omega^2 = \frac{MG}{r^3} \quad -(21)$$

This is Kepler's third law, obtained from the force equation

$$m(\ddot{r} - r\dot{\phi}^2) = -\frac{mMG}{r^2} \quad -(22)$$

when:

$$\ddot{r} = 0, \quad \omega = \dot{\phi} \quad -(23)$$

From (2) in (1)

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 (d\phi + \omega dt)^2 \quad -(24)$$

so the Newtonian velocity is changed to:

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\phi}{dt}\right)^2 + 2r^2\omega \frac{d\phi}{dt} + r^2\omega^2$$

$$= \left(\frac{dr}{dt}\right)^2 + r^2 \frac{(d\phi + \omega dt)^2}{dt^2} \quad -(25)$$

Therefore the frame rotation augments the Newtonian velocity follows:

$$V_1^2 = V_N^2 + V_T^2 + 2 r^2 \omega \frac{d\phi}{dt} \quad - (26)$$

, produces the planetary precession:

$$\Delta \phi_T = 2\pi \left(\left(1 - \frac{V_T^2}{c^2} \right)^{-1/2} - 1 \right) \quad - (27)$$

In the solar system, $\Delta \phi_T$ is very tiny, and:

$$V_1 \sim V_N \quad - (28)$$

o an excellent approximation.

Interpretation of β and β_0

The results of UFT408 are obtained with the interpretation:

$$\beta = \gamma - 1 = \left(1 - \frac{V_N^2}{c^2} \right)^{-1/2} - 1 \quad - (29)$$

nd

$$\beta_0 = \frac{1}{2} \frac{V_N^2}{c^2} \quad - (30)$$

) planetary precession is described with the interpretation:

$$\Delta \phi_T = 2\pi\beta \quad - (31)$$

t

$$\beta = \left(1 - \frac{V_T^2}{c^2} \right)^{-1/2} - 1 \quad - (32)$$

The Lorentz factor in the presence of Thomas rotation becomes:

$$\gamma_1 = \left(1 - \frac{V_1^2}{c^2} \right)^{-1/2} \quad - (33)$$

Since planetary precession is always observed, e Lorentz factor is generalized to Eq. (33).

Therefore the fundamental ECE2 covariant quantities all generalized, by replacing γ by γ_1. For example the [relat]ivistic velocity is:

$$\underline{v} = \gamma_1 \underline{v}_1 \quad -(34)$$

$$v^2 = \frac{v_1^2}{1 - \frac{v_1^2}{c^2}} \quad -(35)$$

[an]d

$$v_1^2 = \frac{v^2}{1 + \frac{v^2}{c^2}} \quad -(36)$$

[T]he Lorentz factor is given by:

$$\gamma_1 = \left(1 + \frac{v^2}{c^2}\right)^{1/2} = \left(1 - \frac{v_1^2}{c^2}\right)^{-1/2} \quad -(37)$$

The Newtonian light deflection due to gravitation becomes:

$$\Delta\phi = \frac{2MG}{R_0 v_1^2} \quad -(38)$$

[T]he velocity of light is c, so:

$$v \rightarrow c \quad -(39)$$

and

$$v_1^2 \rightarrow \frac{c^2}{2} \quad -(40)$$

giving the exactly correct experimental result:

$$\Delta\phi = \frac{4MG}{R_0 c^2} \quad -(41)$$

The hamiltonian becomes:

$$H = \gamma_1 mc^2 + U, \quad -(42)$$

[th]e lagrangian becomes:

$$\mathcal{L} = -\frac{mc^2}{\gamma_1} - U \quad -(43)$$

[an]d so on.