406(7): Final Version of Note 406(6)

The basic equation being considered is:

$$\underline{F} = m \frac{d}{dt}\left(\gamma \underline{v}_N\right) = -\underline{\nabla}\,\phi_0 + \underline{\omega}\,\phi_0 \quad -(1)$$

$$= m \frac{d\underline{v}}{dt}$$

where

$$\underline{v} = \gamma \underline{v}_N \quad -(2)$$

is the Newtonian velocity $\underline{v}_N$ multiplied by the Lorentz factor γ. This is known as the relativistic velocity. From eq. (2):

$$v^2 = \frac{v_N^2}{1 - \frac{v_N^2}{c^2}} \quad -(3)$$

and

$$v_N^2 = \frac{v^2}{1 + \frac{v^2}{c^2}} \quad -(4)$$

$$\xrightarrow[v \to c]{} \frac{c^2}{2}$$

Eq. (4) produces the observed light deflection due to gravity:

$$\Delta\phi = \frac{4MG}{c^2 R_0} \quad -(5)$$

from the Newtonian result:

$$\Delta\phi_N = \frac{2MG}{v_N^2 R_0} \quad -(6)$$

As the relativistic velocity v approaches the speed of light it follows that:

$$\frac{d\underline{v}}{dt} \to \underline{0} \quad -(7)$$

because c is taken to be a universal constant. So for a beam of light:

where

$$\underline{F} = \underline{0} = -\underline{\nabla}\phi_0 + \underline{\omega}\phi_0 \quad -(8)$$

$$\phi_0 = -\frac{mMG}{r} \quad -(9)$$

So

$$\omega_r = -\frac{1}{r} \quad -(10)$$

If the starting equation (1) is taken to be:

$$\underline{F} = m\frac{d}{dt}\left(\gamma \underline{v}_N\right) = -\underline{\nabla}\phi_0 - \underline{\omega}\phi_0 \quad -(11)$$

then at the speed of light:

$$\omega_r = \frac{1}{r} \quad -(12)$$

Since

$$\omega = \frac{2}{3}\frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^3} \quad -(13)$$

the use of a positive sign for ω_r is indicated in order to obtain:

$$\frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^2} = \frac{3}{2} \quad -(14)$$

This is taken to be the maximum value attained by $\langle \delta \underline{r} \cdot \delta \underline{r} \rangle$. At the speed of light, very large vacuum fluctuations occur. In the Newtonian limit on the other hand, there are no vacuum fluctuations.

In previous work it was shown that the equation:

$$\gamma^3 \ddot{\underline{r}} = -MG\frac{\underline{r}}{r^3} \quad -(15)$$

produces a precessing ellipse. In the presence of a spin connection eq. (15) is modified to:

$$\gamma^3 \ddot{\underline{r}} = -MG\frac{\underline{r}}{r^3} + \frac{\phi_0}{m}\underline{\omega}$$

$$= -MG\frac{\underline{r}}{r^3} - \frac{MG}{r}\underline{\omega} \quad -(16)$$

3)

$$= -\frac{mG}{r}\left(\frac{\underline{r}}{r^2} + \underline{\omega}\right)$$

For the radial component:

$$\gamma^3{}_r = -\frac{mG}{r}\left(\frac{1}{r} + \omega_r\right) \quad -(17)$$

$$= -\frac{mG}{r}\left(\frac{1}{r} + \frac{2}{3}\frac{\langle \delta\underline{r} \cdot \delta\underline{r}\rangle}{r^3}\right)$$

This gives an equation for $\langle \delta\underline{r} \cdot \delta\underline{r}\rangle$ in general.