.05(3): The Geodetic Precession as a Thomas Precession in a Rotating Frame.

Consider the infinitesimal line element of E(E) theory:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 \quad -(1)$$

plane polar coordinates. The Thomas precession results

from

$$\phi \rightarrow \phi + \omega t \quad -(2)$$

so

$$ds^2 = \left(1 - \frac{v^2}{c^2}\right)\left(c^2 dt^2 - 2\Omega r^2 d\phi\, dt\right) - dr^2 \quad -(3)$$

where

$$v = \omega r \quad -(4)$$

and

$$\Omega = \omega \left(1 - \frac{v^2}{c^2}\right)^{-1} \quad -(5)$$

Define

$$dt' := \left(1 - \frac{v^2}{c^2}\right)^{1/2} dt \quad -(6)$$

and the Thomas precession is:

$$\Delta\alpha = \Omega\, dt' - \omega\, dt \quad -(7)$$

For a 2π rotation:

$$\omega\, dt = 2\pi \quad -(8)$$

so

$$\Omega\, dt' = 2\pi \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(9)$$

and

$$\Delta\alpha = 2\pi \left(\left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1\right) \quad -(10)$$

If

$$v \ll c: \quad -(11)$$

$$\Delta\alpha \sim \frac{\pi v^2}{c^2} \quad -(12)$$

For example, the precession of a satellite such as gravity probe B as observed from the centre of the earth is given by eq. (12), where v is the velocity of the observer frame with origin at the centre of the earth.

Gravity Probe B has a nearly circular orbit so its Newtonian velocity around the Earth is:

$$v_N^2 = \frac{MG}{} \quad -(13)$$

It is claimed experimentally that the geodetic precession of Gravity Probe B is (PRL 106, 221101): $-(14)$

$$\Delta\phi = 6,601.8 \text{ milliarcseconds per year}$$

where

$$1 \text{ milliarcsecond} = 4.848 \times 10^{-9} \text{ radians} \quad -(15)$$

and

$$\text{one year} = 365 \times 24 \times 60 \text{ minutes} \quad -(16)$$

Gravity Probe B orbited in 90 minutes, so

$$\Delta\phi\,(\text{experimental}) = 5.48 \times 10^{-9} \text{ radians per orbit.} \quad -(17)$$

From eqs. (12) and (17):

$$v^2 = \frac{c^2}{\pi} \times 5.48 \times 10^{-9} \; m^2 s^{-2} \quad -(18)$$

$$= 1.568 \times 10^{8} \; m^2 s^{-2}$$

From eq. (13)

$$v_N^2 = 0.568 \times 10^{8} \; m^2 s^{-2} \quad -(19)$$

So

$$\left(\frac{v}{v_N}\right)^2 = 2.76 \quad -(20)$$

3) The experimental claim is reproduced exactly by the Thomas velocity of:

$$v = 1.252 \times 10^4 \ m s^{-1} \quad -(21)$$

in the ECE2 covariant theory.

This is a simple and straightforward result of ECE2 theory.

The obsolete standard model describes the de Sitter or geodetic precession as the rotation of the so-called "Schwarzschild metric":

$$ds^2 = c^2 d\tau^2 = a c^2 dt^2 - \frac{1}{a} dr^2 - r^2 d\phi^2 \quad -(22)$$

where:

$$a = \left(1 - \frac{2mG}{rc^2}\right) \quad -(23)$$

The rotation is described by eq. (2) and produces:

$$ds^2 = ac^2 - r^2\left(d\phi^2 + 2\omega \, d\phi \, dt + \omega^2 dt^2\right) - \frac{1}{a} dr^2$$

$$= \left(1 - \frac{1}{c^2}\left(v^2 + \frac{2mG}{r}\right)\right)\left(c^2 dt^2 - 2\Omega r^2 d\phi \, dt\right) - \left(1 - \frac{2mG}{rc^2}\right)^{-1} dr^2 \quad -(24)$$

So the geodetic precession according to this de Sitter theory is:

$$\Delta \phi_g = 2\pi\left(\left(1 - \frac{v_1^2}{c^2}\right)^{-1/2} - 1\right) \quad -(25)$$

$$\sim \pi \frac{v_1^2}{c^2}$$

where $$v_1^2 = v^2 + \frac{2MG}{r} \quad -(26)$$

$$= v^2 + 2v_N^2$$

where v is the velocity defined in eq. (4). This velocity is the same for the Thomas and de Sitter theories.

To reproduce the claimed experimental result (H):

$$v^2 = v_1^2 - 2v_N^2$$

$$= 1.568 \times 10^8 - 0.568 \times 10^8$$

$$= 1.00 \times 10^8 \quad -(27)$$

Summary

If the geodetic precession is described by eqs. (1) to (3), the velocity of frame rotation is:

$$v = 1.252 \times 10^4 \text{ m s}^{-1} \quad -(28)$$

If the geodetic precession is described by the ECE2 eq. (22), together with eq. (4), the velocity of frame rotation is

$$v = 1.00 \times 10^4 \text{ m s}^{-1} \quad -(29)$$

The ECE2 is preferred because it is simpler and covariant. It takes account of both torsion and curvature.

Note that the Lense Thirring effect is claimed to be:

$$\Delta\phi_{LT} = 39.2 \text{ milliarcseconds per year} \quad -(30)$$

$$= 3.25 \times 10^{-11} \text{ radians per orbit}$$

which agrees well with the result of Note 405(1):

$$\Delta\phi_{LT}(\text{ECE2}) = 4.10 \times 10^{-11} \text{ radians per orbit} \quad -(31)$$

$$= \frac{2}{5} \frac{MGR^2\omega_E}{c^2 r^3}$$