Proof of Eq. (16) of UFT 142

To Prove

$$q^{\nu}{}_{b}\,\partial_\mu q^{a}{}_{\nu} = \partial_\mu q^{a}{}_{b} \quad -(1)$$

Proof Consider the tetrad postulate:

$$D_\mu q^{a}{}_{\nu} = \partial_\mu q^{a}{}_{\nu} + \omega^{a}{}_{\mu b}\, q^{b}{}_{\nu} - \Gamma^{\lambda}{}_{\mu\nu}\, q^{a}{}_{\lambda} \quad -(2)$$

$$= 0$$

Using the rules given by Sean Carroll in chapter 3:

$$\omega^{a}{}_{\mu b}\, q^{b}{}_{\nu} = \omega^{a}{}_{\mu\nu} \quad -(3)$$

and

$$\Gamma^{\lambda}{}_{\mu\nu}\, q^{a}{}_{\lambda} = \Gamma^{a}{}_{\mu\nu} \quad -(4)$$

so eq. (2) becomes:

$$\partial_\mu q^{a}{}_{\nu} = \Gamma^{a}{}_{\mu\nu} - \omega^{a}{}_{\mu\nu} \quad -(5)$$

From eqs (1) and (5):

$$q^{\nu}{}_{b}\,\partial_\mu q^{a}{}_{\nu} = q^{\nu}{}_{b}\left(\Gamma^{a}{}_{\mu\nu} - \omega^{a}{}_{\mu\nu}\right)$$

$$= \Gamma^{a}{}_{\mu b} - \omega^{a}{}_{\mu b} \quad -(6)$$

Now write eq. (5) with

$$\nu = b \quad -(7)$$

then

$$\partial_\mu q^{a}{}_{b} = \Gamma^{a}{}_{\mu b} - \omega^{a}{}_{\mu b} \quad -(8)$$

From eqs. (6) and (8):

$$q^{\nu}{}_{b}\,\partial_\mu q^{a}{}_{\nu} = \partial_\mu q^{a}{}_{b} \quad -(9)$$

which is eq. (1), Q.E.D.