

```
(%i1) kill(all);
(%o0) done
```

1 Eq. (9)

```
(%i1) depends(drdr, r);
(%o1) [drdr(r)]
```

```
(%i2) omega: 2/3*drdr/r^3;
(%o2) 
$$\frac{2 \operatorname{drdr}}{3 r^3}$$

```

```
(%i3) Delta_phi: r^2/2*(omega/r-diff(omega,r));
(%o3) 
$$\frac{\left( \frac{8 \operatorname{drdr}}{3 r^4} - \frac{2 \left( \frac{d}{d r} \operatorname{drdr} \right)}{3 r^3} \right) r^2}{2}$$

```

```
(%i4) expand(%);
(%o4) 
$$\frac{4 \operatorname{drdr}}{3 r^2} - \frac{\frac{d}{d r} \operatorname{drdr}}{3 r}$$

```

2 Eq. (40)

```
(%i5) assume(r>0, L>0);
(%o5) [r>0, L>0]
```

```
(%i6) f: 1/r^3*sqrt(L^2*r^2-L_r^2);
(%o6) 
$$\frac{\sqrt{L^2 r^2 - L_r^2}}{r^3}$$

```

```
(%i7) omega_0: -r*v/(2*m[E]*c^2)*diff(f,r);
(%o7) 
$$-\frac{r \left( \frac{d}{d r} \frac{\sqrt{L^2 r^2 - L_r^2}}{r^3} \right) v}{2 m_E c^2}$$

```

```
(%i8) omega_1: ev(omega_0, [L_r=0]), diff;
(%o8) 
$$\frac{L v}{m_E c^2 r^2}$$

```

3 Eq. (42)

```
(%i9) kill(omega);
(%o9) done
```

```
(%i10) omega_2: ev(omega_1, [L=2/5*m[E]*r[E]^2*omega[E]]);
(%o10) 
$$\frac{2 \operatorname{omega}_E r_E^2 v}{5 c^2 r^2}$$

```

□ 4 Eq. (48)

```
(%i11) depends([omega,v],r);
(%o11) [omega(r),v(r)]
```

```
(%i12) Delta_phi: r^2/2*(omega_1/r-diff(omega,r));
(%o12) 
$$\frac{r^2 \left( \frac{L v}{m_E c^2 r^3} - \frac{d}{d r} \operatorname{omega} \right)}{2}$$

```

```
(%i13) omega: -v*L/(m[E]*c^2*r^2);
(%o13) 
$$-\frac{L v}{m_E c^2 r^2}$$

```

```
(%i14) diff(omega,r);
(%o14) 
$$\frac{2 L v}{m_E c^2 r^3} - \frac{L \left( \frac{d}{d r} v \right)}{m_E c^2 r^2}$$

```

```
(%i15) (ev(Delta_phi,diff));
(%o15) 
$$\frac{r^2 \left( \frac{L \left( \frac{d}{d r} v \right)}{m_E c^2 r^2} - \frac{L v}{m_E c^2 r^3} \right)}{2}$$

```

```
(%i16) Delta_phi_1: factor(ev(Delta_phi,diff));
(%o16) 
$$\frac{L \left( r \left( \frac{d}{d r} v \right) - v \right)}{2 m_E c^2 r}$$

```

□ 4.1 alternative

```
(%i17) omega: v*L/(m[E]*c^2*r^2);
(%o17) 
$$\frac{L v}{m_E c^2 r^2}$$

```

```
(%i18) diff(omega,r);
(%o18) 
$$\frac{L \left( \frac{d}{d r} v \right)}{m_E c^2 r^2} - \frac{2 L v}{m_E c^2 r^3}$$

```

```
(%i19) (ev(Delta_phi,diff));
```

$$r^2 \frac{\left(\frac{3 L v}{m_E c^2 r^3} - \frac{L \left(\frac{d}{d r} v \right)}{m_E c^2 r^2} \right)}{2}$$

```
(%o19)
```

```
(%i20) Delta_phi_1a: factor(ev(Delta_phi,diff));
```

$$-\frac{L \left(r \left(\frac{d}{d r} v \right) - 3 v \right)}{2 m_E c^2 r}$$

```
(%o20)
```

5 Eq. (51)

```
(%i21) Delta_phi_2: ev(Delta_phi_1a, [diff(v,r)=0]);
```

$$\frac{3 L v}{2 m_E c^2 r}$$

```
(%o21)
```

```
(%i22) kill(omega,v);
```

```
(%o22) done
```

```
(%i23) L: 2/5*m[E]*r[E]^2*omega[E];
```

$$\frac{2 m_E \omega_E r_E^2}{5}$$

```
(%o23)
```

```
(%i24) Delta_phi_3: expand(ev(Delta_phi_2));
```

$$\frac{3 \omega_E r_E^2 v}{5 c^2 r}$$

```
(%o24)
```

6 Numerical evaluation

```
(%i25) v1: omega[E]*r[E];
```

$$\omega_E r_E$$

```
(%o25)
```

```
(%i26) v2: sqrt(m[E]*G/r);
```

$$\frac{\sqrt{m_E G}}{\sqrt{r}}$$

```
(%o26)
```

6.1 Evaluation

```
(%i27) c: 2.99792458e8;
```

$$2.99792458 \cdot 10^8$$

```
(%o27)
```

```
(%i28) r[E]: 6.371009e6;
```

$$6371009.0$$

```
(%o28)
```

```

[ (%i29)  G: 6.67384e-11;
[ (%o29)  6.67384 10-11

[ (%i30)  m[E]: 5.97219e24;
[ (%o30)  5.97219 1024

[ (%i31)  omega[E]: 7.292115e-5;
[ (%o31)  7.292115 10-5

[ Orbital radius of Gravity Probe B

[ (%i32)  r: 7.02e6;
[ (%o32)  7020000.0

[ (%i33)  ev(L);
[ (%o33)  7.070718599544239 1033

[ (%i35)  v1: ev(v1);
          v2: ev(v2);
[ (%o34)  464.58130294035
[ (%o35)  7535.050151319897

[ (%i37)  ev(omega_2, [v=v1]);
          ev(omega_2, [v=v2]);
[ (%o36)  1.24186971173432 10-19
[ (%o37)  2.01419008903265 10-18

[ (%i39)  ev(Delta_phi_3, [v=v1]);
          ev(Delta_phi_3, [v=v2]);
[ (%o38)  1.307688806456239 10-12
[ (%o39)  2.120942163751381 10-11

```