```
(%i1)  kill(all);
(%o0)  done
```

1 Eq. (1)

1.1 with variable omega(u)

```
(%i1)  depends([u,r],phi);
(%o1)  [u(φ), r(φ)]
```

```
(%i2)  E1: diff(u,phi,2)+u=omega(u)/(1+epsilon*cos(phi));
(%o2)  d²/dφ² u + u = ω(u)/(ϵ cos(φ) + 1)
```

```
(%i3)  ode2(E1,u,phi);
(%o3)  false
```

1.2 with constant omega_u

```
(%i4)  E1: diff(u,phi,2)+u=omega_u/(1+epsilon*cos(phi));
(%o4)  d²/dφ² u + u = ω_u/(ϵ cos(φ) + 1)
```

```
(%i5)  assume(epsilon^2-1<0);
(%o5)  [ϵ² < 1]
```

```
(%i6)  ode2(E1,u,phi);
(%o6)  u = (√(1 − ϵ²) (2 ω_u sin(φ) atan(sin(φ)/(cos(φ) + 1)) + ω_u cos(φ) log(ϵ cos(φ) + 1)) − 2 ω_u sin(φ)
atan(√(1 − ϵ²) sin(φ)/((ϵ + 1) cos(φ) + ϵ + 1))) / (ϵ √(1 − ϵ²)) + %k1 sin(φ) + %k2 cos(φ)
```

2 Eq. (2)

```
(%i7)  E2: diff(u,phi,2)+u=1/alpha*(1+omega_u/u);
(%o7)  d²/dφ² u + u = (ω_u/u + 1)/α
```

```
(%i8)  ode2(E2,u,phi);
```

$$(\%o8)\quad \left[-\int \frac{1}{\sqrt{\frac{2\,\omega_u\log(u)-\alpha\,u^2+2\,u-2\,\%k1}{\alpha}}}\,du=\varphi+\%k2,\ \int \frac{1}{\sqrt{\frac{2\,\omega_u\log(u)-\alpha\,u^2+2\,u-2\,\%k1}{\alpha}}}\,du=\varphi+\%k2\right]$$

3 Simplifying the integral

3.1 Omitting the log function

```
(%i9)  integrate(1/sqrt((-alpha*u^2+2*u-2*%k1)/alpha),u);
Is 2 %k1 α-1 zero or nonzero? n;
```

$$(\%o9)\quad -\operatorname{asin}\left(\frac{\frac{2}{\alpha}-2\,u}{\sqrt{\frac{4}{\alpha^2}-\frac{8\,\%k1}{\alpha}}}\right)$$

3.2 Omitting the linear and quadratic u functions

```
(%i10) integrate(1/sqrt((2*omega_u*log(u)-2*%k1)/alpha),u);
```

$$(\%o10)\quad \frac{\sqrt{\pi}\,\sqrt{\alpha}\,\%e^{\frac{\%k1}{\omega_u}}\,\operatorname{erf}\left(\frac{\sqrt{-\frac{\alpha}{\omega_u}}\,\sqrt{\frac{2\,\omega_u\log(u)-2\,\%k1}{\alpha}}}{\sqrt{2}}\right)}{\sqrt{2}\,\sqrt{-\frac{\alpha}{\omega_u}}\,\omega_u}$$

4 Eq. (5)

```
(%i11) E5: diff(u,phi,2)+u=omega_u*(1-epsilon*cos(phi));
```

$$(\%o11)\quad \frac{d^2}{d\,\varphi^2}\,u+u=\omega_u\left(1-\epsilon\cos(\varphi)\right)$$

```
(%i12) res: ode2(E5,u,phi);
```

$$(\%o12)\quad u=-\frac{\epsilon^2\,\omega_u\,\varphi\sin(\varphi)+\left(\epsilon^2+1\right)\omega_u\cos(\varphi)-2\,\epsilon\,\omega_u}{2\,\epsilon}+\%k1\sin(\varphi)+\%k2\cos(\varphi)$$

```
(%i13) res1: expand(ev(res,[phi=%pi/2, %k1=0]));
```

$$(\%o13)\quad u=\omega_u-\frac{\pi\,\epsilon\,\omega_u}{4}$$