Trace of the Gamma Connection

The trace(s) of the gamma connection in tensorial notation can be written (summed over repeated indices)

$$\Gamma^{\rho}_{\mu\mu} = \Gamma^{\rho}_{\mu\nu}\ \delta^{\nu}_{\mu} = 0 \tag{1}$$

$$\Gamma^{\mu}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu}\ \delta^{\mu}_{\rho} = 0 \tag{2}$$

$$\Gamma^{\nu}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu}\ \delta^{\nu}_{\rho} = 0 \tag{3}$$

The sums are all assumed to be zero following the lead of paper 298., but this needs to be proven. This represents twelve equations, four each for of equations (1) through (3). They are not all independent, for example. if one applies the antisymmetry of $\Gamma^{\rho}_{\mu\nu}$ to equation (3), we have

$$\Gamma^{\nu}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu}\ \delta^{\nu}_{\rho} = -\Gamma^{\rho}_{\nu\mu}\,\delta^{\nu}_{\rho} = -\Gamma^{\nu}_{\nu\mu} \tag{4}$$

Equations (2) and (3) are the same (since it doesn't matter what the summation index is called). It is interesting to note the antisymmetry of the two traces $\Gamma^{\nu}_{\mu\nu}$ and $\Gamma^{\nu}_{\nu\mu}$.