95(4): Maclaurin Series Method for Considering the Effect of the Vacuum.

Consider the Maclaurin series for a function $f(x)$:

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^n}{n!} f^{(n)}(0) \quad -(1)$$

and consider any scalar function f of $\underline{r} + \delta\underline{r}$, where $\delta\underline{r}$ is a vacuum fluctuation. It follows that:

$$f(\underline{r} + \delta\underline{r}) = f(\underline{r}) + (\underline{r} + \delta\underline{r}) \cdot \underline{\nabla} f + \frac{1}{2} (\underline{r} + \delta\underline{r})^2 \nabla^2 f + \dots \quad -(2)$$

At

$$\underline{r} = \underline{0} \quad -(3)$$

$$\Delta f = f(\delta\underline{r}) - f(\underline{0})$$
$$= \delta\underline{r} \cdot \underline{\nabla} f(\underline{r}) + \frac{1}{2} (\delta\underline{r} \cdot \underline{\nabla})^2 f(\underline{r}) + \frac{1}{3} (\delta\underline{r} \cdot \underline{\nabla})^3 f(\underline{r}) + \dots \quad -(3)$$

On isotropic averaging in the vacuum:

$$\langle \delta\underline{r} \rangle = \underline{0} \quad -(4)$$

and

$$\langle (\delta\underline{r} \cdot \underline{\nabla})^2 \rangle = \left\langle \left(\delta X \frac{\partial}{\partial X} + \delta Y \frac{\partial}{\partial Y} + \delta Z \frac{\partial}{\partial Z} \right)^2 \right\rangle \quad -(5)$$

Use:

$$\langle \delta X^2 \rangle = \langle \delta Y^2 \rangle = \langle \delta Z^2 \rangle = \frac{1}{3} \langle (\delta r)^2 \rangle \quad -(6)$$

and

$$\langle \delta X \delta Z \rangle = \langle \delta X \delta Y \rangle = \langle \delta Y \delta Z \rangle = 0 \quad -(7)$$

2)

So for any scalar function f:

$$\boxed{\langle \Delta f \rangle = \frac{1}{6} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle \nabla^2 f} \quad -(8)$$

is the average change in f due to the vacuum.

Eq. (8) can be used for any scalar potential ϕ, and for components of any vector function such as $\underline{A}$, $\underline{B}$ and $\underline{E}$.

Therefore for example:

$$\langle \Delta \phi \rangle = \frac{1}{6} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle \nabla^2 \phi \quad -(9)$$

and

$$\langle \Delta A_x \rangle = \frac{1}{6} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle \nabla^2 A_x \quad -(10)$$

$$\langle \Delta A_y \rangle = \frac{1}{6} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle \nabla^2 A_y \quad -(11)$$

$$\langle \Delta A_z \rangle = \frac{1}{6} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle \nabla^2 A_z \quad -(12)$$

so

$$\langle \Delta \underline{A} \rangle = \langle \Delta A_x \rangle \underline{i} + \langle \Delta A_y \rangle \underline{j} + \langle \Delta A_z \rangle \underline{k}$$
$$= \frac{1}{6} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle \left(\nabla^2 A_x \underline{i} + \nabla^2 A_y \underline{j} + \nabla^2 A_z \underline{k} \right) \quad -(13)$$

Eq. (13) is also true for the electric field strength $\underline{E}$ and the magnetic flux density $\underline{B}$, and in any system of coordinates.

The change in potential responsible for the Lamb shift in atomic H is:

$$\langle \Delta \phi \rangle = \frac{1}{6} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle \left\langle \nabla^2 \left(-\frac{e^2}{4\pi \epsilon_0 r} \right) \right\rangle \quad -(14)$$

where the expectation value is:

$$\left\langle \nabla^2 \left(\frac{-e^2}{4\pi \epsilon_0 r} \right) \right\rangle = -\frac{e^2}{4\pi \epsilon_0} \int \psi^*(\underline{r}) \nabla^2 \left(\frac{1}{r} \right) \psi(\underline{r}) d\underline{r}$$

$$= \frac{e^2}{\epsilon_0} | \psi(0) |^2 \quad -(15)$$

where we have used the Dirac delta function:

$$\nabla^2 \left(\frac{1}{r} \right) = -4\pi \delta(\underline{r}) \quad -(16)$$

This is problematic, because differentiation gives:

$$\nabla^2 \left(\frac{1}{r} \right) = 0 \quad -(17)$$

but the Dirac delta function gives:

$$\psi_{2s}(0) = \frac{1}{(8\pi a_0^3)^{1/2}} \quad -(18)$$

for the H atom, and $\psi_{2p}(0) = 0. \quad -(19)$

Using mode theory:

$$\langle \delta \underline{r} \cdot \delta \underline{r} \rangle \sim \frac{1}{2\epsilon_0 \pi^2} \frac{e^2}{\hbar c} \left(\frac{\hbar}{mc} \right)^2 \log_e \frac{4\epsilon_0 \hbar c}{e^2} \quad -(20)$$

so the change in potential energy due to the Lamb shift in atomic H is:

$$\langle \Delta \phi \rangle = \frac{\alpha^5 mc^2}{6\pi} \log_e \frac{1}{\pi \alpha} \quad -(20)$$

where α is the fine structure constant. This is about 1.0 GHz, very similar to the observed shift.

) This calculation can be repeated for any $\underline{A}$, $\underline{B}$ and $\underline{E}$ and the spin connections found. Eq. (20) can be taken to be a general property of the vacuum, it affects any measurable quantity in physics.

Examples

Magnetic Dipole Potential

In this case:

$$\underline{A} = \frac{\mu_0}{4\pi} \frac{\underline{m} \times \underline{r}}{|\underline{r}|^3} \quad -(21)$$

and $\langle \Delta \underline{A} \rangle$ is found from Eq. (13).

Magnetic Dipole Field

$$\underline{B} = \frac{\mu_0}{4\pi r^3} \left(\frac{3\underline{r}(\underline{m} \cdot \underline{r})}{r^2} - \underline{m} \right) \quad -(22)$$

and $\langle \Delta \underline{B} \rangle$ is found from Eq. (13). In ECE2 theory

$$\underline{B} = \underline{B}_0 - \underline{\omega} \times \underline{A}_0 \quad -(23)$$

so

$$\langle \Delta \underline{B} \rangle = \underline{B} - \underline{B}_0 = - \underline{\omega} \times \underline{A}_0 \quad -(24)$$

and the spin connection may be found.

Coulomb Electric Field

$$\underline{E} = - \frac{e}{4\pi \epsilon_0 r^2} \underline{e}_r \quad -(25)$$

and $\langle \Delta \underline{E} \rangle$ is found from eq. (13).

) Gravitational Field

$$\underline{g} = -\frac{mG}{r^2}\,\underline{e}_r \quad -(26)$$

and $\langle \Delta \underline{g} \rangle$ is found from eq. (13).

When considering the vacuum electric field $\underline{E}_{vac}$ then:

$$\underline{F} = m\frac{d^2}{dt^2}(\delta \underline{r})_{vac} = -e\underline{E}_{vac} \quad -(27)$$

and this force may be considered to be the driving force for Euler Bernoulli resonance in a circuit. So the vacuum effects may be greatly amplified by a particular circuit design.