394(8): Effect of the Vacuum on the Contact Term of NMR

By computer algebra:

$$\underline{\nabla}\left(\frac{1}{r}\right) = -\frac{\underline{r}}{r^3} \quad -(1)$$

$$= -\frac{\left(X\underline{i} + Y\underline{j} + Z\underline{k}\right)}{\left(x^2+y^2+z^2\right)^{3/2}}$$

and

$$\nabla^2\left(\frac{1}{r}\right) = -\frac{3}{r^3} + 3\frac{\underline{r}\cdot\underline{r}}{r^5} = 0 \quad -(2)$$

However, in contact theory:

$$\nabla^2\left(\frac{1}{r}\right) = -4\pi\,\delta_D(r) \quad -(3)$$

where

$$\delta_D(r) \neq 0 \quad -(4)$$

is the Dirac delta function. The latter is also used in quantum mechanics, but clearly, the contradiction between (2) and (3) means that $\delta_D(r)$ cannot be interpreted simply. At first, mathematicians dismissed the δ_D function as complete nonsense, but later, a new type of mathematics was developed to explain it.

In this note we bypass these difficulties with the m theory and the change of coordinate:

$$\underline{r} \rightarrow \underline{r} + \delta\underline{r} \quad -(5)$$

$$= \underline{r}_1$$

Eq. (1) becomes:

$$\underline{\nabla}\left(\frac{1}{|\underline{r}+\delta\underline{r}|}\right) = -\frac{(\underline{r}+\delta\underline{r})}{|\underline{r}+\delta\underline{r}|^3} \quad -(6)$$

) As in previous work:

$$|\underline{r} + \delta \underline{r}|^3 = \left(r^2 + 2\underline{r} \cdot \delta \underline{r} + \delta \underline{r} \cdot \delta \underline{r} \right)^{3/2}$$

$$= r^3 (1+x)^{3/2} \quad - (7)$$

where $x = \frac{1}{r^2} \left(2\underline{r} \cdot \delta \underline{r} + \delta \underline{r} \cdot \delta \underline{r} \right) \quad - (8)$

so:

$$\underline{\nabla} \left(\frac{1}{|\underline{r} + \delta \underline{r}|} \right) = - \frac{(\underline{r} + \delta \underline{r})}{r^3 (1+x)^{3/2}}$$

$$= - \frac{1}{r^3} (\underline{r} + \delta \underline{r}) \left(1 - \frac{3x}{2} + \frac{15}{8} x^2 + \ldots \right) \quad - (9)$$

The isotropic average of this quantity can be evaluated using these methods, eq (2) in the presence of the vacuum becomes:

$$\nabla^2 \left(\frac{1}{r_1} \right) = - \frac{3}{r_1^3} + \frac{3\, \underline{r}_1 \cdot \underline{r}_1}{r_1^5} \quad - (10)$$

$$= - \frac{3}{r^3 (1+x)^{3/2}} + \frac{3 (\underline{r} + \delta \underline{r}) \cdot (\underline{r} + \delta \underline{r})}{r^5 (1+x)^{5/2}}$$

$$= - \frac{3}{r^3} \left(1 - \frac{3x}{2} + \frac{15}{8} x^2 + \ldots \right)$$

$$+ \frac{3 (\underline{r} + \delta \underline{r}) \cdot (\underline{r} + \delta \underline{r})}{r^5} \left(1 - \frac{5x}{2} + \frac{35}{8} x^2 + \ldots \right)$$

$$\neq 0$$

The isotropic average $\left\langle \nabla^2 \left(\frac{1}{r_1} \right) \right\rangle$ can be worked out by computer and the contact field is:

$$\underline{B}\,(\text{contact}) = -\frac{\mu_0}{4\pi}\, m \left\langle \nabla^2 \left(\frac{1}{r_1} \right) \right\rangle \quad -(11)$$

$$\neq 0$$

This can be worked out to any order in x.

Finally a ~~theory~~ of $\delta \underline{r}$ is used to model the effect on the NMR spectrum.