

94(5): Note a Covariant of Vector Antisymmetry

In MZ they, the magnetic flux density is calculated from:

$$\langle B \rangle_{ij} = (\nabla \times \underline{A} - \langle \underline{\omega} \rangle \times \underline{A})_{ij} \quad - (1)$$

$$= - \langle B \rangle_{ji}$$

So:

$$(\partial_1 + \omega_1) A_2 = - (\partial_2 + \omega_2) A_1 \quad - (2)$$

$$(\partial_2 + \omega_2) A_3 = - (\partial_3 + \omega_3) A_2 \quad - (3)$$

$$(\partial_1 + \omega_1) A_3 = - (\partial_3 + \omega_3) A_1 \quad - (4)$$

Write eq. (2) as:

$$\partial_1 A_2 + \omega_1 A_2 + \partial_2 A_1 + \omega_2 A_1 = 0 \quad - (5)$$

Transforming from covariant to Cartesian:

$$-\frac{\partial A_y}{\partial x} + \omega_x A_y - \frac{\partial A_x}{\partial y} + \omega_y A_x = 0 \quad - (6)$$

i.e.

$$\frac{\partial A_x}{\partial y} + \frac{\partial A_y}{\partial x} = \omega_x A_y + \omega_y A_x \quad - (7)$$

et cyclicum

Q.E.D.
 Eq. (7) is the vector antisymmetry law used in previous work. It is equivalent to:

$$\langle B \rangle_{xy} = - \langle B \rangle_{yx} \quad - (8)$$

in Cartesian, and in covariant:

$$\langle B \rangle_{ij} = - \langle B \rangle_{ji} \quad - (9)$$

In MZ they, $\langle \underline{B} \rangle$ and \underline{A} are known and $\langle \underline{\omega} \rangle$ is calculated from them using eq. (1), where antisymmetry is obeyed automatically.