1 Average value calclation for Macroscopic Zitter

```
(%i1)  kill(all);
(%o0)  done
```

```
(%i1)  /* Function for averaging delta r terms in an expression */
       averageMZ(f, dr) :=
       block([arglist, s0, den, i, j, res, l, el],
       /* make list of all summands in the formula */
       arglist: [],
       s0: string(0),
       arglist: args(expand(f)),
       /* check if dr[i] appears in any denominator */
       for j:1 thru length(arglist) do (
           den: denom(arglist[j]),
           /*print ("DEN:", den),*/
           for i:1 thru length(dr) do (
               if not freeof(dr[i],den) then print(i, "Denominator error!")
       )),
       /* remove all list elements containing occurences of dr[i], dr[i]^3,
       for j:1 thru length(arglist) do (
           for i:1 thru length(dr) do (
               if not equal(string(coeff(arglist[j], dr[i], 1)), s0)
               or not equal(string(coeff(arglist[j], dr[i], 3)), s0)
               or not equal(string(coeff(arglist[j], dr[i], 5)), s0)
               then
                   arglist[j]: 0
                   /*print("L(", dr[i], ") ", arglist[j])*/
       )),
       res: 0,
       /* construct result expression */
       l: length(arglist),
       for j:1 thru l do (
           el: pop(arglist),
           res: res + el
           /*print(j,el,res)*/
       ),
       res
       /*print(res)*/
       )$
```

```
(%i2)  /* Function for replacing deltaX^2 etc. to delta r^2 terms
           in cartesian coordinates */
       replaceR(f,dri) :=
           block([f1, i, dr, len],
           f1: f,
           len: length(dri),
           for i:1 thru len do (
               f1: ratsubst(1/len*dR[av]^2, dri[i]^2, f1)
           ),
           factorsum(f1)
       )$
```

2 <B> contact term

```
(%i5)  r: [X, Y, Z];
       dr: [dX, dY, dZ];
       r_1: sqrt(X^2+Y^2+Z^2);
(%o3)  [X, Y, Z]
(%o4)  [dX, dY, dZ]
(%o5)  sqrt(Z^2+Y^2+X^2)
```

```
(%i6)  x: 1/r_1^2*(2*r.dr + dr.dr);
(%o6)  (dZ^2+2 (Z dZ+Y dY+X dX)+dY^2+dX^2)/(Z^2+Y^2+X^2)
```

```
(%i7)  assume (r_0>0);
(%o7)  [r_0>0]
```

```
(%i8)  m: [m_X, m_Y, m_Z];
(%o8)  [m_X, m_Y, m_Z]
```

2.1 1st order approximation in x

```
(%i9)  f: -3/r_1^3*(1-3*x/2)
       +3/r_1^5*(r+dr).(r+dr)*(1-5*x/2);
(%o9)
3 (1-5 (dZ^2+2 (Z dZ+Y dY+X dX)+dY^2+dX^2)/(2 (Z^2+Y^2+X^2))) ((dZ+Z)^2+(dY+Y)^2+(dX+X)^2)
-------------------------------------------------------------------------------------------
                                        (Z^2+Y^2+X^2)^(5/2)
 3 (1-3 (dZ^2+2 (Z dZ+Y dY+X dX)+dY^2+dX^2)/(2 (Z^2+Y^2+X^2)))
-------------------------------------------------------------- -
                        (Z^2+Y^2+X^2)^(3/2)
```

```
(%i11)  f1: averageMZ((f), dr);
        f2: replaceR(f1,dr);
```

(%o10)

$$-\frac{15\,dZ^4}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{5/2}}-\frac{30\,dY^2\,dZ^2}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{5/2}}-\frac{30\,dX^2\,dZ^2}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{5/2}}-\frac{75\,Z^2\,dZ^2}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{5/2}}-\frac{15\,Y^2\,dZ^2}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{5/2}}-\frac{15\,X^2\,dZ^2}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{5/2}}+\frac{9\,dZ^2}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{3/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{3/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{3/2}}+\frac{3\,dZ^2}{\left(Z^2+Y^2+X^2\right)^{5/2}}-\frac{15\,dY^4}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{5/2}}-\frac{30\,dX^2\,dY^2}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{5/2}}-\frac{15\,Z^2\,dY^2}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{5/2}}-\frac{75\,Y^2\,dY^2}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{5/2}}-\frac{15\,X^2\,dY^2}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{5/2}}+\frac{9\,dY^2}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{3/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{3/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{3/2}}+\frac{3\,dY^2}{\left(Z^2+Y^2+X^2\right)^{5/2}}-\frac{15\,dX^4}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{5/2}}-\frac{15\,Z^2\,dX^2}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{5/2}}-\frac{15\,Y^2\,dX^2}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{5/2}}-\frac{75\,X^2\,dX^2}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{5/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{5/2}}+\frac{9\,dX^2}{2\,Z^2\left(Z^2+Y^2+X^2\right)^{3/2}+2\,Y^2\left(Z^2+Y^2+X^2\right)^{3/2}+2\,X^2\left(Z^2+Y^2+X^2\right)^{3/2}}+\frac{3\,dX^2}{\left(Z^2+Y^2+X^2\right)^{5/2}}-\frac{3}{\left(Z^2+Y^2+X^2\right)^{3/2}}+\frac{3\,Z^2}{\left(Z^2+Y^2+X^2\right)^{5/2}}+\frac{3\,Y^2}{\left(Z^2+Y^2+X^2\right)^{5/2}}+\frac{3\,X^2}{\left(Z^2+Y^2+X^2\right)^{5/2}}$$

(%o11)

$$-\frac{5\,dR_{av}^2\left(3\,dR_{av}^2+4\left(Z^2+Y^2+X^2\right)\right)}{\left(\right)^{7/2}}$$

(%i12) `Bcontact: facsum(%,dR[av]);`

(%o12)
$$\frac{-15\,dR_{av}^4-20\left(Z^2+Y^2+X^2\right)dR_{av}^2}{2\left(Z^2+Y^2+X^2\right)^{7/2}}$$

(%i13) `expand(Bcontact);`

(%o13)
$$-\frac{15\,dR_{av}^4}{2\left(Z^2+Y^2+X^2\right)^{7/2}}-\frac{10\,Z^2\,dR_{av}^2}{\left(Z^2+Y^2+X^2\right)^{7/2}}-\frac{10\,Y^2\,dR_{av}^2}{\left(Z^2+Y^2+X^2\right)^{7/2}}-\frac{10\,X^2\,dR_{av}^2}{\left(Z^2+Y^2+X^2\right)^{7/2}}$$

(%i14) `factor(%);`

(%o14)
$$-\frac{5\,dR_{av}^2\left(3\,dR_{av}^2+4\,Z^2+4\,Y^2+4\,X^2\right)}{2\left(Z^2+Y^2+X^2\right)^{7/2}}$$

(%i15) `f3: facsum(f2,dR[av]);`

(%o15)
$$\frac{-15\,dR_{av}^4-20\left(Z^2+Y^2+X^2\right)dR_{av}^2}{2\left(Z^2+Y^2+X^2\right)^{7/2}}$$

(%i16) `f4: ratsubst(r_0^2, X^2+Y^2+Z^2, f3);`

(%o16)
$$-\frac{20\,dR_{av}^2\,r_0{}^2+15\,dR_{av}^4}{2\,r_0{}^7}$$

(%i17) `f5: expandwrt(f4, dR[av]);`

(%o17)
$$-\frac{10\,dR_{av}^2}{r_0{}^5}-\frac{15\,dR_{av}^4}{2\,r_0{}^7}$$

(%i18) `f6: box(expandwrt(factor(f5), dR[av]));`

(%o18)
$$-\frac{10\,dR_{av}^2}{r_0{}^5}-\frac{15\,dR_{av}^4}{2\,r_0{}^7}$$

(%i19) `factor(%);`

(%o19)
$$-\frac{5\,dR_{av}^2\left(4\,r_0{}^2+3\,dR_{av}^2\right)}{2\,r_0{}^7}$$

2.2 2nd order approximation in x

(%i20)
```
f: -3/r_1^3*(1-3*x/2+15*x^2/8)
+3/r_1^5*(r+dr).(r+dr)*(1-5*x/2+35*x^2/8)$
```

(%i22)
```
f1: averageMZ(f, dr)$
f2: replaceR(f1,dr)$
```

(%i23)
```
f3: facsum(f2,dR[av]);
```
(%o23)
$$\frac{105\,dR_{av}^6+420\left(Z^2+Y^2+X^2\right)dR_{av}^4}{8\left(Z^2+Y^2+X^2\right)^{9/2}}$$

(%i24)
```
f4: ratsubst(r_0^2, X^2+Y^2+Z^2, f3);
```
(%o24)
$$\frac{420\,dR_{av}^4\,r_0{}^2+105\,dR_{av}^6}{8\,r_0{}^9}$$

(%i25)
```
f5: expandwrt(f4, dR[av]);
```
(%o25)
$$\frac{105\,dR_{av}^4}{2\,r_0{}^7}+\frac{105\,dR_{av}^6}{8\,r_0{}^9}$$

(%i26)
```
f6: box(expandwrt(factor(f5), dR[av]));
```
(%o26)
$$\frac{105\,dR_{av}^4}{2\,r_0{}^7}+\frac{105\,dR_{av}^6}{8\,r_0{}^9}$$

(%i27)
```
factor(%);
```
(%o27)
$$\frac{105\,dR_{av}^4\left(4\,r_0{}^2+dR_{av}^2\right)}{8\,r_0{}^9}$$