3(5): Ensemble Average for ~~Q~~ Dipole Potential

To first order in x:

$$\langle \phi \rangle = \frac{1}{4\pi\epsilon_0 r^3} \left\langle (\underline{r} + \delta\underline{r}) \cdot \underline{p} \left(1 - \frac{3x}{2}\right) \right\rangle \quad -(1)$$

where

$$x = \frac{1}{r^2} \left(2\underline{r} \cdot \delta\underline{r} + \delta\underline{r} \cdot \delta\underline{r} \right) \quad -(2)$$

So:

$$\langle \phi \rangle = \frac{1}{4\pi\epsilon_0 r^3} \left(\underline{r} \cdot \underline{p} + \langle \delta\underline{r} \cdot \underline{p} \rangle \right) - \frac{3\underline{p}}{8\pi\epsilon_0 r^5} \cdot \left\langle (\underline{r} + \delta\underline{r})(2\underline{r} \cdot \delta\underline{r} + \delta\underline{r} \cdot \delta\underline{r}) \right\rangle \quad -(3)$$

By isotropy: $\langle \delta\underline{r} \cdot \underline{p} \rangle = 0 \quad -(4)$

In eq. (3):

$$\left\langle (\underline{r} + \delta\underline{r})(2\underline{r} \cdot \delta\underline{r} + \delta\underline{r} \cdot \delta\underline{r}) \right\rangle$$

$$= \underline{i} \left\langle (x + \delta x)(2x\delta x + 2y\delta y + 2z\delta z + \delta x^2 + \delta y^2 + \delta z^2) \right\rangle + \dots \quad -(5)$$

By isotropy:

$$\langle \delta x \delta y \rangle = \langle \delta x \delta z \rangle = \langle \delta y \delta z \rangle = 0 \quad -(6)$$

so

$$\left\langle (\underline{r} + \delta\underline{r})(2\underline{r} \cdot \delta\underline{r} + \delta\underline{r} \cdot \delta\underline{r}) \right\rangle = 3\underline{r} \langle \delta\underline{r} \cdot \delta\underline{r} \rangle \quad -(7)$$

Therefore to first order in x:

$$\boxed{\langle \phi \rangle = \frac{\underline{p} \cdot \underline{r}}{4\pi \epsilon_0 r^3} \left(1 - \frac{9}{2r^2} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle \right)} \quad -(8)$$

The standard model dipole potential is:

$$\phi = \frac{\underline{p} \cdot \underline{r}}{4\pi \epsilon_0 r^3} \quad -(9)$$

As in the theory of the Lamb shift in H, the vacuum contributes a term in $\langle \delta \underline{r} \cdot \delta \underline{r} \rangle$. The true dipole potential is always eq. (8) to first order in x. This is because the vacuum is always present. This means that the radiative corrections are always present, and conclusively demonstrate the fact that the vacuum creates potentials and fields in matter and in circuits.

The vacuum changes the r dependence of the dipole potential. To first order in x it becomes the sum of terms in $1/r^3$ and $1/r^5$.

The ensemble averaged electric field strength due to $\langle \phi \rangle$ will be calculated in the next note to first order in x.