3(4): Derivation of the Dipole Electric Field from the Dipole Potential: Shivering induced by the Vacuum.

The standard model dipole potential is well known to be:

$$\phi_0 = \frac{1}{4\pi\epsilon_0 r^3} \underline{r} \cdot \underline{p} \quad -(1)$$

where:

$$\underline{r} = \underline{x} - \underline{x}_0 \quad -(2)$$

$$r = | \underline{x} - \underline{x}_0 | . \quad -(3)$$

and the dipole moment $\underline{p}$ is an intrinsic property of the charge distribution. The electric field strength $\underline{E}$ at point $\underline{x}$ due to a dipole moment $\underline{p}$ at point $\underline{x}_0$ is:

$$\underline{E}_0 = -\underline{\nabla} \phi_0 \quad -(4)$$

Therefore:

$$\underline{E}_0 = -\frac{1}{4\pi\epsilon_0} \underline{\nabla} \left(\frac{\underline{r} \cdot \underline{p}}{r^3} \right). \quad -(5)$$

In Cartesian coordinates:

$$\underline{E}_0 = -\frac{1}{4\pi\epsilon_0} \underline{\nabla} \left(\frac{X p_X + Y p_Y + Z p_Z}{(X^2+Y^2+Z^2)^{3/2}} \right)$$

$$= -\frac{1}{4\pi\epsilon_0} \frac{\partial}{\partial X} \left(\frac{X p_X + Y p_Y + Z p_Z}{(X^2+Y^2+Z^2)^{3/2}} \right) \underline{i} + \ldots \quad -(6)$$

The dipole components p_X, p_Y and p_Z have no dependence on X, so:

$$\underline{E}_0 = -\frac{p_X}{4\pi\epsilon_0} \frac{\partial}{\partial X} \left(\frac{X}{(X^2+Y^2+Z^2)^{3/2}} \right) \underline{i} + \ldots$$

$$= -\frac{p_X \underline{i}}{4\pi\epsilon_0} \left(\frac{1}{(X^2+Y^2+Z^2)^{3/2}} + X \frac{\partial}{\partial X} \left(\frac{1}{(X^2+Y^2+Z^2)^{3/2}} \right) \right)$$

$$= -\frac{p_x}{4\pi\epsilon_0}\,\underline{i}\left(\frac{1}{(X^2+Y^2+Z^2)^{3/2}} - \frac{3X}{(X^2+Y^2+Z^2)^{5/2}}\right)$$

$$+ \cdots$$

$$= \frac{1}{4\pi\epsilon_0}\,\underline{i}\left(\frac{3p_x X}{(X^2+Y^2+Z^2)^{5/2}} - \frac{1}{(X^2+Y^2+Z^2)^{3/2}}\right) + \cdots$$

$$-(7)$$

Adding the $\underline{j}$ and $\underline{k}$ terms the complete result is:

$$\underline{E}_0 = \frac{1}{4\pi\epsilon_0 r^3}\left(\frac{3\underline{r}(\underline{p}\cdot\underline{r})}{r^2} - \underline{p}\right) -(8)$$

where

$$\underline{r} = \underline{x} - \underline{x}_0 \quad -(9)$$

and

$$r = |\underline{x} - \underline{x}_0| \quad -(10)$$

Eq. (8) is the electric field strength at point $\underline{x}$ due to a dipole moment $\underline{p}$ at point $\underline{x}_0$.

Shivering or Zitterbewegung due to the Vacuum.

The vacuum is not considered in standard classical electrodynamics but it is considered in quantum mechanics to explain the Lamb shift of atomic H.

We adopt the general law that the vacuum produces:

$$\boxed{\underline{r} \rightarrow \underline{r} + \delta\underline{r}} \quad -(11)$$

wherever $\underline{r}$ occurs. Here $\delta\underline{r}$ is the shivering term,

*) originally inferred by Schroedinger in 1930 from the Dirac equation, and named "zitterbewegung" or "shivering".
The position vector is defined by:

$$\underline{r} = X\underline{i} + Y\underline{j} + Z\underline{k} \quad -(12)$$

so

$$X \rightarrow X + \delta X \quad -(13)$$

$$Y \rightarrow Y + \delta Y \quad -(14)$$

$$Z \rightarrow Z + \delta Z \quad -(15)$$

The effect of the vacuum on the $\underline{\nabla}$ operator is therefore:

$$\underline{\nabla} \rightarrow \frac{\partial}{\partial(X+\delta X)}\underline{i} + \frac{\partial}{\partial(Y+\delta Y)}\underline{j} + \frac{\partial}{\partial(Z+\delta Z)}\underline{k} \quad -(16)$$

The dipole moment $\underline{p}$ has no dependence on X, Y and Z so is not affected by the vacuum. The dipole moment $\underline{p}$ is an intrinsic property of a molecule, listed in standard tables.

Therefore the effect of the vacuum on the dipole potential is to change it to:

$$\phi = \frac{1}{4\pi\epsilon_0 |\underline{r}+\delta\underline{r}|^3}(\underline{r}+\delta\underline{r})\cdot\underline{p} \quad -(17)$$

The effect of the vacuum is:

$$\Delta\phi(\mathrm{vac}) = \frac{\delta\underline{r}\cdot\underline{p}}{4\pi\epsilon_0 |\underline{r}+\delta\underline{r}|^3} \quad -(18)$$

In eq. (18):

4)

$$|\underline{r}+\delta\underline{r}| = \left((\underline{r}+\delta\underline{r})\cdot(\underline{r}+\delta\underline{r})\right)^{1/2}$$

$$= \left(r^2 + 2\underline{r}\cdot\delta\underline{r} + \delta\underline{r}\cdot\delta\underline{r}\right)^{1/2}$$

$$= r\left(1 + 2\frac{\underline{r}\cdot\delta\underline{r}}{r^2} + \frac{\delta\underline{r}\cdot\delta\underline{r}}{r^2}\right)^{1/2} \quad -(19)$$

So

$$|\underline{r}+\delta\underline{r}|^3 = r^3\left(1+2\frac{\underline{r}\cdot\delta\underline{r}}{r^2} + \frac{\delta\underline{r}\cdot\delta\underline{r}}{r^2}\right)^{3/2} \quad -(20)$$

Therefore:

$$\boxed{\Delta\phi(\text{vac}) = \frac{\delta\underline{r}\cdot\underline{p}}{4\pi\epsilon_0 r^3\left(1+2\frac{\underline{r}\cdot\delta\underline{r}}{r^2}+\frac{\delta\underline{r}\cdot\delta\underline{r}}{r^2}\right)^{3/2}}} \quad -(21)$$

The complete dipole potential in the presence of the vacuum is:

$$\phi = \frac{(\underline{r}+\delta\underline{r})\cdot\underline{p}}{4\pi\epsilon_0 r^3\left(1+2\frac{\underline{r}\cdot\delta\underline{r}}{r^2}+\frac{\delta\underline{r}\cdot\delta\underline{r}}{r^2}\right)^{3/2}}$$

$$= \frac{(\underline{r}+\delta\underline{r})\cdot\underline{p}}{4\pi\epsilon_0|\underline{r}+\delta\underline{r}|^3} \quad -(22)$$

From eq. (8), the electric field strength in the presence of the vacuum is:

$$\boxed{\underline{E} = \frac{1}{4\pi\epsilon_0 |\underline{r}+\delta\underline{r}|^3}\left(\frac{(\underline{r}+\delta\underline{r})(\underline{p}\cdot(\underline{r}+\delta\underline{r}))}{|\underline{r}+\delta\underline{r}|^2} - \underline{p}\right)} \quad -(23)$$

Therefore:

$$\underline{E} = \frac{1}{4\pi\epsilon_0}\left(\frac{(\underline{r}+\delta\underline{r})(\underline{p}\cdot(\underline{r}+\delta\underline{r}))}{|\underline{r}+\delta\underline{r}|^5} - \frac{\underline{p}}{|\underline{r}+\delta\underline{r}|^3}\right) \quad -(24)$$

in which:

$$|\underline{r}+\delta\underline{r}|^5 = r^5\left(1 + \frac{2\underline{r}\cdot\delta\underline{r}}{r^2} + \frac{\delta\underline{r}\cdot\delta\underline{r}}{r^2}\right)^{5/2} \quad -(25)$$

and

$$|\underline{r}+\delta\underline{r}|^3 = r^3\left(1 + 2\frac{\underline{r}\cdot\delta\underline{r}}{r^2} + \frac{\delta\underline{r}\cdot\delta\underline{r}}{r^2}\right)^{3/2} \quad -(26)$$

By conservation of scalar antisymmetry:

$$\underline{E} = -\underline{\nabla}\phi_0 + \underline{\omega}\phi_0 \quad -(27)$$

$$= -\frac{\partial \underline{A}_0}{\partial t} - \omega_0 \underline{A}_0$$

$$= -\underline{\nabla}\phi$$

in which $\underline{A}_0$ is electric vector potential in the hypothetical absence of the vacuum, and where:

$$\omega^\mu = \left(\frac{\omega_0}{c}, \underline{\omega}\right) \quad -(28)$$

) is the spin connection four vector of ECE2 unified field theory.

Therefore:

$$\boxed{\underline{E} = \underline{E}_0 + \underline{\omega}\,\phi_0} \quad - (29)$$

is the electric field in the presence of the vacuum. The vacuum is always present, so $\underline{E}$ is always the measured electric field.

Ensemble Averaged Field and Potential

Denote:

$$x := \frac{1}{r^2}\left(2\,\underline{r}\cdot\delta\underline{r} + \delta\underline{r}\cdot\delta\underline{r}\right) \quad - (30)$$

then

$$\phi = \frac{(\underline{r} + \delta\underline{r})\cdot\underline{p}}{4\pi\epsilon_0 r^3 (1+x)^{3/2}} \quad - (31)$$

and

$$\underline{E} = \frac{1}{4\pi\epsilon_0}\left(\frac{(\underline{r}+\delta\underline{r})(\underline{p}\cdot(\underline{r}+\delta\underline{r}))}{r^5(1+x)^{5/2}} - \frac{\underline{p}}{r^3(1+x)^{3/2}}\right)$$

$$- (32)$$

Note carefully that ϕ and $\underline{E}$ are always the measured dipole potential and field.

As in the well known and well tested theory of the Lamb shift in H, ensemble averages

7) of the fluctuations $\delta \underline{r}$ are used. The vacuum is made up of constantly fluctuating $\delta \underline{r}$. In an isotropic vacuum:

$$\langle \delta \underline{r} \rangle = \underline{0} \quad -(33)$$

but

$$\langle \delta \underline{r} \cdot \delta \underline{r} \rangle \neq 0. \quad -(34)$$

For:

$$x \ll 1 \quad -(35)$$

the binomial expansion gives:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \dots \quad -(36)$$

so

$$(1+x)^{-3/2} = 1 - \frac{3x}{2} + \frac{15}{8} x^2 + \dots \quad -(37)$$

and

$$(1+x)^{-5/2} = 1 - \frac{5x}{2} + \frac{35}{8} x^2 + \dots \quad (38)$$

It follows that:

$$\phi \sim \frac{(\underline{r} + \delta \underline{r}) \cdot \underline{p}}{4\pi \epsilon_0 r^3} \left(1 - \frac{3x}{2} + \frac{15x^2}{8} + \dots \right) \quad -(39)$$

and

$$\underline{E} = \frac{1}{4\pi \epsilon_0} \left(\frac{(\underline{r} + \delta \underline{r})(\underline{p} \cdot (\underline{r} + \delta \underline{r}))}{r^5} \left(1 - \frac{5x}{2} + \frac{35}{8} x^2 + \dots \right) - \frac{\underline{p}}{r^3} \left(1 - \frac{3x}{2} + \frac{15x^2}{8} + \dots \right) \right) \quad -(40)$$

The ensemble averages $\langle \phi \rangle$ and $\langle \underline{E} \rangle$ will be worked out in the next note.