393(3): Extension of Zitterbewegung Theory to Electric and Magnetic Dipole Fields (4FT385).

The general rule is postulated in electrodynamics that:

$$r \rightarrow r + \delta r \quad - (1)$$

where δr represents the shivering motion of the electron induced by the vacuum. As in 4FT385, the electric dipole field in the absence of δr is:

$$\underline{E}_0(\underline{r}) = \frac{3\underline{n}(\underline{p}\cdot\underline{n}) - \underline{p}}{4\pi\epsilon_0 |\underline{r} - \underline{r}_0|^3}, \quad - (2)$$

which in the dipole approximation becomes:

$$\underline{E}_0 = \frac{p}{4\pi\epsilon_0 r^3}\left(2\cos\theta\, \underline{e}_r + \sin\theta\, \underline{e}_\theta\right) \quad - (3)$$

This is the hypothetical field in the absence of the vacuum. The vacuum is never absent, so Eq. (3) for the shivering electron becomes:

$$\underline{E} = \frac{p}{4\pi\epsilon_0 (r + \delta r)^3}\left(2\cos\theta\, \underline{e}_r + \sin\theta\, \underline{e}_\theta\right) \quad - (4)$$

Eq. (4) can be expressed as:

$$\underline{E} = \frac{p}{4\pi\epsilon_0 r^3 \left(1 + \frac{\delta r}{r}\right)^3}\left(2\cos\theta\, \underline{e}_r + \sin\theta\, \underline{e}_\theta\right). \quad - (5)$$

If

$$\delta r \ll r \quad - (6)$$

) the binomial expansion can be used:

$$\left(1+x\right)^{-3} = 1 + \frac{1}{3}x + \frac{2}{9}x^2 + \frac{14}{81}x^3 + \ldots \quad -(7)$$

where

$$x = \frac{\delta r}{r} \quad -(8)$$

Therefore:

$$\underline{E} = \frac{p}{4\pi\epsilon_0 r^3}\left(1 + \frac{1}{3}x + \frac{2}{9}x^2 + \frac{14}{81}x^3 + \ldots\right)\left(2\cos\theta\,\underline{e}_r + \sin\theta\,\underline{e}_\theta\right) \quad -(9)$$

In the isotropic assumption:

$$\langle \delta\underline{r} \rangle = \underline{0} \quad -(10)$$

where $\langle\ \rangle$ denotes ensemble averaging. So

$$\langle \underline{E} \rangle = \underline{E}_0 + \frac{p}{18\pi\epsilon_0}\frac{\langle \Delta r^2 \rangle}{r^5}\left(2\cos\theta\,\underline{e}_r + \sin\theta\,\underline{e}_\theta\right) + \ldots \quad -(11)$$

The mean square displacement can be evaluated with a suitable vacuum theory as in note 393(2). The general expression (2) in the presence of the vacuum becomes:

$$\underline{E}(\underline{r}) = \frac{3\underline{n}(\underline{p}\cdot\underline{n}) - \underline{p}}{4\pi\epsilon_0\left|\underline{r} - \underline{r}_0 + \delta(\underline{r} - \underline{r}_0)\right|} \quad -(12)$$

) and the general rule (1) is extended to:

$$\underline{r} - \underline{r_0} \rightarrow \underline{r} - \underline{r_0} + \delta(\underline{r} - \underline{r_0}) \quad -(13)$$

The most general electrostatic field for the shivering electron is:

$$\underline{E} = -\frac{1}{4\pi\epsilon_0}\underline{\nabla}\int\frac{\rho(\underline{x}')}{|\underline{x}-\underline{x_0}+\delta(\underline{x}-\underline{x_0})|}d^3x' \quad -(14)$$

The most general magnetostatic field for the shivering electron is:

$$\underline{B} = \frac{\mu_0}{4\pi}\underline{\nabla}\times\int\frac{\underline{J}(\underline{x}')}{|\underline{x}-\underline{x_0}+\delta(\underline{x}-\underline{x_0})|}d^3x' \quad -(15)$$

$$:= \underline{\nabla}\times\underline{A},$$

so

$$\underline{A} = \frac{\mu_0}{4\pi}\int\frac{\underline{J}(\underline{x}')}{|\underline{x}-\underline{x_0}+\delta(\underline{x}-\underline{x_0})|}d^3x' \quad -(16)$$

In the dipole approximation:

$$\underline{B} = \frac{\mu_0}{4\pi(r+\delta r)^3}(I\pi a^2)\left(\cos\theta\,\underline{e}_r + \sin\theta\,\underline{e}_\theta\right) \quad -(17)$$

$$= \underline{B}_0 + \frac{\mu_0}{4\pi}(I\pi a^2)\left(\cos\theta\,\underline{e}_r + \sin\theta\,\underline{e}_\theta\right)\frac{\langle\Delta r^2\rangle}{r^5}$$

where the hypothetical magnetic flux density is

4) the absence of the vacuum is:

$$\underline{B}_0 = \frac{\mu_0}{4\pi r^3} \left(I \pi a^2 \right) \left(\cos\theta \, \underline{e}_r + \sin\theta \, \underline{e}_\theta \right) \quad -(18)$$

Therefore the graphs of UFT385 are supplemented by a term proportional to $\langle \Delta r^2 \rangle / r^5$.

The electrostatic and magnetostatic spin connection vectors are worked out with:

$$\underline{E} = - \underline{\nabla} \phi = - \underline{\nabla} \phi_0 + \underline{\omega} \phi_0 \quad -(19)$$

where the Coulombic potential is

$$\phi_0(\underline{x}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\underline{x}')}{|\underline{x} - \underline{x}'|} d^3x' \quad -(20)$$

Defining the electric dipole moment by:

$$\underline{p} = \int \underline{x}' \rho(\underline{x}') d^3x' \quad -(21)$$

the dipole potential may be defined as:

$$\phi_0(\underline{x}) = \frac{1}{4\pi\epsilon_0} \frac{\underline{p} \cdot \underline{x}}{r^3} \quad -(22)$$

So:

$$\underline{E}_0(\underline{x}) = - \underline{\nabla} \phi_0(\underline{x})$$

$$= \frac{3\underline{n}(\underline{p} \cdot \underline{n}) - \underline{p}}{4\pi\epsilon_0 |\underline{x} - \underline{x}_0|^3} \quad -(23)$$

In the presence of the vacuum the electron

) or macroscopic charge shiver, so:

$$\phi(\underline{x}) = \frac{1}{4\pi\epsilon_0} \frac{\underline{p}\cdot(\underline{x}+\delta\underline{x})}{(r+\delta r)^3} \quad -(24)$$

So:

$$\underline{E}(\text{shivering}) = \underline{E}_0 + \underline{\omega}\,\phi_0 \quad -(25)$$

$$= \frac{3\underline{n}(\underline{p}\cdot\underline{n})-\underline{p}}{4\pi\epsilon_0|\underline{x}-\underline{x}_0|^3} + \frac{1}{4\pi\epsilon_0}\frac{\underline{p}\cdot\underline{x}}{r^3}\underline{\omega}$$

$$= \frac{3\underline{n}(\underline{p}\cdot\underline{n})-\underline{p}}{4\pi\epsilon_0|\underline{x}-\underline{x}_0+\delta(\underline{x}-\underline{x}_0)|^3} \quad -(26)$$

This assumes that the dipole moment is not affected by the shivering. Since vacuum effects are small in the zitterbewegung theory, this is a good approximation.