393(2): Electric Field in the Presence of the Vacuum and Conservation of Antisymmetry

The electric field in the presence of the vacuum is, by conservation of scalar antisymmetry:

$$\underline{E} = -\underline{\nabla}\phi_0 + \underline{\omega}\phi_0 = -\frac{\partial \underline{A}_E}{\partial t} - \omega_0 \underline{A}_E \quad -(1)$$

where ϕ_0 is the Coulomb potential:

$$\phi_0 = \frac{-e}{4\pi\epsilon_0 r_0}, \quad -(2)$$

and where $\underline{A}_E$ is the electric vector potential. The spin connection four vector is:

$$\omega^\mu = \left(\frac{\omega_0}{c}, \underline{\omega}\right) \quad -(3)$$

From note 393(1):

$$\langle \underline{\omega} \rangle = 3\frac{\langle \delta r^2 \rangle}{r_0^3} \underline{e}_r \quad -(4)$$

where

$$r = r_0 + \delta r. \quad -(5)$$

Here δr represents the shivering or zitterbewegung of the electron due to the influence of the vacuum. Eq. (5) is known to lead to an accurate description of the Lamb shift.

From eqs. (1) and (4):

$$\underline{E} = -\frac{e}{4\pi\epsilon_0 r_0^2}\underline{e}_r - \frac{3\langle \delta r^2 \rangle e}{4\pi\epsilon_0 r_0^4}\underline{e}_r \quad -(6)$$

So the electric field strength in the presence of the vacuum is:

$$\underline{E} = -\frac{e}{4\pi\epsilon_0 r_0^2}\left(1 + \frac{3\langle \delta r^2\rangle}{r_0^2}\right)\underline{e}_r \qquad -(7)$$

$$= -\frac{\partial \underline{A}_E}{\partial t} - \omega_0 \underline{A}_E$$

By conservation of trace antisymmetry in electrostatics

$$\frac{\partial \phi_0}{\partial t} + \omega_0 \phi_0 = 0 \qquad -(8)$$

where ϕ_0 is the Coulomb potential, the potential in the hypothetical absence of the vacuum. The Coulomb potential by definition is for static charge, so

$$\frac{\partial \phi_0}{\partial t} = 0. \qquad -(9)$$

It follows that:

$$\omega_0 = 0. \qquad -(10)$$

for the Coulomb potential. From eqns (7) and (10):

$$\frac{\partial \underline{A}_E}{\partial t} = \frac{e}{4\pi\epsilon_0 r_0^2}\left(1 + \frac{3\langle \delta r^2\rangle}{r_0^2}\right)\underline{e}_r \qquad (11)$$

$$= -\underline{E}$$

Note carefully that the concept of $\underline{A}_E$ does

3) does not exist in the standard model.

In the theory of the Lamb shift in atomic H, $\underline{E}$ in eq. (7) is the fluctuating electric field between the electron and the proton, and $\langle \delta r^2 \rangle$ is worked out from the wave functions. Using mode theory, the classical equation of motion for $(\delta \underline{r})_k$ induced by a single mode of the field of wave vector $\underline{k}$ and frequency ω is:

$$m \frac{d^2}{dt^2} (\delta \underline{r})_k = - e \underline{E}_k \quad - (12)$$

For a field oscillating at ω:

$$\delta \underline{r}(t) \sim \delta r(0) \left(e^{-i\omega t} + e^{i\omega t} \right) \quad - (13)$$

So

$$(\delta \underline{r})_{\underline{k}} \sim \frac{e}{mc^2 k^2} \underline{E}_k \quad - (14)$$

$$= \frac{e}{mc^2 k^2} \mathcal{E}_{\underline{k}} \left(\underline{a}_k e^{-i\omega t} + \ldots \right)$$

where

$$\mathcal{E}_{\underline{k}} = \left(\frac{\hbar c k}{2 \epsilon_0 \Omega} \right)^{1/2} \quad - (15)$$

where Ω is a large normalization volume.

By summing over all $\underline{k}$:

$$\langle (\delta \underline{r})^2 \rangle = \sum_k \left(\frac{e}{mc^2 k^2} \right)^2 \langle 0 | \underline{E}_k | 0 \rangle$$

$$= \sum_{\underline{k}} \left(\frac{e}{mc^2 k^2} \right)^2 \left(\frac{\hbar c k}{2 \epsilon_0 \Omega} \right)$$

4)

$$= \frac{2\Omega}{(2\pi)^3} 4\pi \int d\kappa\, \kappa^2 \left(\frac{\hbar}{mc\kappa^2}\right)^2 \left(\frac{\hbar c\kappa}{2\epsilon_0 \Omega}\right) \quad -(16)$$

This follows because continuity of $\underline{\kappa}$ implies:

$$\sum_{\kappa} \rightarrow \frac{2\Omega}{(2\pi)^3} \int d^3\kappa \quad -(17)$$

It follows that

$$\langle (\delta \underline{r})^2 \rangle = \frac{1}{2\epsilon_0 \pi^2} \frac{e^2}{\hbar c} \left(\frac{\hbar}{mc}\right)^2 \int \frac{d\kappa}{\kappa} \quad -(18)$$

and for an isotropic sample (i.e. vacuum):

$$\langle \delta \underline{r} \rangle = 0 \quad -(19)$$

In the usual model the upper and lower limits of the integral are chosen to obtain convergence. This gives:

$$\langle \delta \underline{r}^2 \rangle = \frac{1}{2\epsilon_0 \pi^2} \left(\frac{e^2}{\hbar c}\right) \left(\frac{\hbar}{mc}\right)^2 \log_e \frac{4\epsilon_0 \hbar c}{e^2} \quad -(20)$$

Therefore the electric field in the presence of the vacuum can be calculated from eqns. (7) and (20). The mean square correlation is calculated from eqns (4) and (20).

It is also possible to calculate the mean square displacement using the self term of the van Hove correlation function:

$$\langle r^2(t) \rangle = \int_0^\infty G_s(\underline{r}, t)\, dr \quad -(21)$$

5) as described in:

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Therefore the van Hove correlation function of vacuum particles can be computed by simulation.

In general, the contribution of the vacuum to a current is given by Eq. (7)
