

3.7.2 (a): Methodology for Two Dimensional Orbits

The scalar antisymmetry law is

$$\underline{g} = -\underline{\nabla} \underline{\Phi} + \underline{\omega} \underline{\Phi} = -\frac{\partial \underline{\Phi}}{\partial t} - \underline{\omega}_0 \underline{\Phi} \quad (1)$$

ii) add the spin connection is:

$$\underline{\omega}^\mu = \left(\frac{\underline{\omega}_0}{c}, \underline{\omega} \right) \quad (2)$$

The material part of \underline{g} is:

$$\underline{g} = -\underline{\nabla} \underline{\Phi} - \frac{\partial \underline{\Phi}}{\partial t} \quad (3)$$

and there is a contribution to \underline{g} from the vacuum:

$$\underline{g}(\text{vacuum}) = \underline{\omega} \underline{\Phi} = -\underline{\omega}_0 \underline{\Phi} \quad (4)$$

Therefore:

$$\underline{\nabla} \underline{\Phi} = \frac{\partial \underline{\Phi}}{\partial t} \quad (5)$$

Therefore:

$$\underline{g} = -\underline{\nabla} \underline{\Phi} = -2\underline{\nabla} \underline{\Phi} \quad (6)$$

The material scalar potential ϕ is defined by:

$$\phi = \int \frac{\rho(\text{matter})}{|\underline{x} - \underline{x}'|} d^3x = -\frac{mG}{r} \quad (7)$$

and is the gravitational potential. Here $\rho(\text{matter})$ is the source mass density.

The gravitomagnetic field is:

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q} - \underline{\omega} \times \underline{Q} \quad (8)$$

The material gravitomagnetic field is:

$$\underline{\Omega}(\text{matter}) = \underline{\nabla} \times \underline{Q} \quad (9)$$

and here is a vacuum contribution:

$$\underline{\Omega}(\text{vacuum}) = -\underline{\omega} \times \underline{Q} \quad (10)$$

From eq. (9):

$$\underline{Q} = \frac{G}{c^2} \int \frac{\underline{J}(\text{matter})}{|\underline{x} - \underline{x}'|} d^3x \quad (11)$$

From eqs. (4), (7) and (11):

$$\underline{\omega} \int \frac{\rho(\text{matter})}{|\underline{x} - \underline{x}'|} d^3x = -\frac{\omega_0}{c^2} \int \frac{\underline{J}(\text{matter})}{|\underline{x} - \underline{x}'|} d^3x \quad (12)$$

Using

$$\epsilon_0 \mu_0 = \frac{1}{c^2} \quad (13)$$

it follows that:

$$\rho(\text{matter}) \underline{\omega} = -\frac{1}{c^2} \underline{J}(\text{matter}) \omega_0 \quad (14)$$

where

$$\underline{J}^4(\text{matter}) = (c\rho(\text{matter}), \underline{J}(\text{matter})) \quad (15)$$

is it:

As in note 389(4), for a forward precession

$$\omega_x = \frac{x}{x^2 + y^2} \left(\frac{1}{\gamma} - 1 \right) - \frac{\dot{x}\dot{y}y + x\dot{x}^2}{\gamma c^2 (x^2 + y^2)} \quad (16)$$

$$\omega_y = \frac{y}{x^2 + y^2} \left(\frac{1}{\gamma} - 1 \right) - \frac{\dot{y}\dot{x}x + y\dot{y}^2}{\gamma c^2 (x^2 + y^2)} \quad (17)$$

which

$$\gamma = \left(1 - \frac{\dot{x}^2 + \dot{y}^2}{c^2} \right)^{-1/2} \quad (18)$$

For retrograde precession:

$$\omega_x = \left(\frac{x}{x^2 + y^2} \right) \left(\frac{1}{y^3} - 1 \right) - (19)$$

$$\omega_y = \left(\frac{y}{x^2 + y^2} \right) \left(\frac{1}{y^3} - 1 \right) - (20)$$

1) Therefore ω_0 can be calculated from eq. (14), knowing $\underline{\omega}$, ρ (matter) and \underline{J} (matter) the mass density of the source and the current of mass density of the source.

2) The vacuum contribution to \underline{g} is calculated from eq. (4).

3) The vacuum contribution to $\underline{\Omega}$ is calculated from eq. (10).

4) The law of conservation of vector entropy in the xy plane is:

$$\frac{\partial Q_x}{\partial y} + \frac{\partial Q_y}{\partial x} = \omega_x Q_y + \omega_y Q_x - (21)$$

so the left hand side can be calculated knowing \underline{Q} and $\underline{\omega}$.

5) The law of conservation of trace entropy is

$$\frac{1}{c^2} \left(\frac{\partial}{\partial t} + \omega_0 \right) \underline{\Phi} = \left(\underline{\nabla} - \underline{\omega} \right) \cdot \underline{Q} - (22)$$

so $\partial \underline{\Phi} / \partial t$ can be calculated knowing ω_0 , $\underline{\omega}$ and \underline{Q} .

b) The second time derivative of Φ can be calculated from:

$$\frac{1}{c^2} \frac{d^2 \Phi}{dt^2} = \underline{\nabla} \cdot (\underline{\omega} \Phi) \quad (23)$$

7) The gravitational vector potential \underline{Q} is found from eq. (5):

$$\frac{d\underline{Q}}{dt} = \underline{\nabla} \Phi = mG \frac{\underline{r}}{r^3}$$

$$= \frac{mG (x \underline{i} + y \underline{j})}{(x^2 + y^2)^{3/2}} \quad (24)$$

Discussion

The Newtonian Φ has been used in eq. (7), but any Φ can be used. The key to the method is eq. (1):

$$\underline{g} = -\underline{\nabla} \Phi (\text{matter}) + \underline{\omega} \Phi (\text{vacuum contribution})$$

$$= -\frac{d\underline{Q}}{dt} (\text{matter}) - \underline{\omega}_0 \underline{Q} (\text{vacuum contribution}) \quad (25)$$

and

$$-\underline{\nabla} \Phi (\text{matter}) = -\frac{d\underline{Q}}{dt} (\text{matter}) \quad (26)$$

$$\underline{\omega} \Phi (\text{vacuum contribution}) = -\underline{\omega}_0 \underline{Q} (\text{vacuum contribution}) \quad (27)$$

This method is also valid in three dimensions. (21)

The vector symmetry law means that in 2-D there is a balance between matter and vacuum

$$5) \left(\frac{\partial Q_x}{\partial t} + \frac{\partial Q_y}{\partial x} \right) (\text{matter}) = (\omega_x Q_y + \omega_y Q_x) (\text{vacuum contribution}) \quad - (28)$$

The violation of trace symmetry law means that there is another type of balance between matter and the vacuum contribution.

$$\left(\frac{1}{c^3} \frac{\partial \Phi}{\partial t} - \underline{\nabla} \cdot \underline{Q} \right) (\text{matter}) \quad - (29)$$

$$= - \left(\frac{1}{c} \omega \cdot \underline{\Phi} + \underline{\omega} \cdot \underline{Q} \right) (\text{contribution from vacuum})$$

Finally, by defining:

$$\underline{g} = - \underline{\nabla} \Phi - \frac{\partial \underline{Q}}{\partial t} \quad - (30)$$

and

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q} \quad - (31)$$

of homogeneous field equations we derived:

$$\underline{\nabla} \cdot \underline{\Omega} = 0 \quad - (31)$$

and

$$\underline{\nabla} \times \underline{g} + \frac{\partial \underline{\Omega}}{\partial t} = \underline{0} \quad - (32)$$

Any gravitational problem can be worked out completely "this way", giving vacuum maps and showing how the vacuum contributes to matter.