392(1): The Laws of Conservation of Antisymmetry in Two Dimensions

In the notation of Note 379(5) the antisymmetry law in general is:

$$(\partial_\mu + \omega_\mu) q_\nu = -(\partial_\nu + \omega_\nu) q_\mu \quad -(1)$$

where

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \underline{\nabla} \right) \quad -(2)$$

$$q_\mu = \left(\frac{\phi}{c}, -\underline{q} \right) \quad -(3)$$

$$\omega_\mu = \left(\frac{\omega_0}{c}, -\underline{\omega} \right) \quad -(4)$$

Scalar Law

$$-\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial \underline{q}}{\partial t} - \omega_0 \underline{q} \quad -(5)$$

Vector Law

$$\frac{\partial q_X}{\partial Y} + \frac{\partial q_Y}{\partial X} = \omega_X q_Y + \omega_Y q_X \quad -(6)$$

Trace or Lindstrom Law for Gravitation

$$\frac{1}{c^2}\left(\frac{\partial}{\partial t} + \omega_0 \right)\phi = (\underline{\nabla} - \underline{\omega}) \cdot \underline{Q} \quad -(7)$$

$$= \frac{\partial Q_X}{\partial X} + \frac{\partial Q_Y}{\partial Y} - \omega_X Q_X - \omega_Y Q_Y$$

where

$$\underline{Q} = Q^{(0)} \underline{q} \quad -(8)$$

The vector law for gravitation is, in 2-D

$$\frac{\partial Q_X}{\partial Y} + \frac{\partial Q_Y}{\partial X} = \omega_X Q_Y + \omega_Y Q_X \quad -(9)$$

Eq. (5) in 2D gives two scalar laws:

$$-\frac{\partial \phi}{\partial X} + \omega_X \phi = -\frac{\partial Q_X}{\partial t} - \omega_0 Q_X \quad -(10)$$

$$-\frac{\partial \phi}{\partial Y} + \omega_Y \phi = -\frac{\partial Q_Y}{\partial t} - \omega_0 Q_Y \quad -(11)$$

The gravitational potential is

$$\phi = -\frac{mG}{r} = -\frac{mG}{(X^2+Y^2)^{1/2}} \quad -(12)$$

The acceleration due to gravity is:

$$\underline{g} = -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial \underline{Q}}{\partial t} - \omega_0 \underline{Q} \quad -(13)$$

and this is found from the 2-D Lagrangian. This gives the spin connection vector $\underline{\omega}$. Eq. (9) gives the vector potential:

$$\underline{Q} = Q_X \underline{i} + Q_Y \underline{j} \quad -(14)$$

Eq. (7) gives ω_0, noting that:

$$\frac{\partial \phi}{\partial t} \neq 0 \quad -(15)$$

where

$$\phi = -\frac{mG}{r(t)} \quad -(16)$$

Finally eq. (5) gives the derivative $\partial \underline{Q}/\partial t$.