```
(%i1)  kill(all);
(%o0)  done
```

1 Calculation of delta_phi

```
(%i1)  assume(G>0, M>0, alpha>0, epsilon>0);
(%o1)  [G>0,M>0,α>0,ϵ>0]
```

```
(%i2)  gamma: 1+v^2/2/c^2+3/8*v^4/c^4+5/16*v^6/c^6;
(%o2)  5 v^6/(16 c^6)+3 v^4/(8 c^4)+v^2/(2 c^2)+1
```

```
(%i3)  v: sqrt(M*G/alpha*(1+epsilon)^2);
(%o3)  sqrt(G) sqrt(M) (ϵ+1)/sqrt(α)
```

```
(%i4)  H_0: -m*M*G/(2*alpha)*(1-epsilon^2)+U;
(%o4)  U-G M (1-ϵ^2) m/(2 α)
```

```
(%i5)  H_r: (gamma-1)*m*c^2-m*M*G/alpha*(1+epsilon*cos(delta_phi));
(%o5)  c^2 m (5 v^6/(16 c^6)+3 v^4/(8 c^4)+v^2/(2 c^2))-G M (cos(delta_phi) ϵ+1) m/α
```

```
(%i6)  E1: H_0=H_r;
(%o6)  U-G M (1-ϵ^2) m/(2 α)=c^2 m (5 v^6/(16 c^6)+3 v^4/(8 c^4)+v^2/(2 c^2))-
G M (cos(delta_phi) ϵ+1) m/α
```

```
(%i7)  solve(E1, cos(delta_phi));
(%o7)  [cos(delta_phi)=
(5 α m v^6+6 α c^2 m v^4+8 α c^4 m v^2+(-8 G M c^4 ϵ^2-8 G M c^4) m-16 U α c^4)/(16 G M c^4 ϵ m)]
```

```
(%i8)  expand(%);
(%o8)  [cos(delta_phi)=5 α v^6/(16 G M c^4 ϵ)+3 α v^4/(8 G M c^2 ϵ)+α v^2/(2 G M ϵ)-U α/(G M ϵ m)-ϵ/2-1/(2 ϵ)]
```

```
(%i9)  cos_delta_phi: ev(rhs(first(%)));
(%o9)  -U α/(G M ϵ m)+5 G^2 M^2 (ϵ+1)^6/(16 α^2 c^4 ϵ)+3 G M (ϵ+1)^4/(8 α c^2 ϵ)+(ϵ+1)^2/(2 ϵ)-ϵ/2-1/(2 ϵ)
```

2 Parameters

```
(%i10)  c: 2.99792458e8;
(%o10)  2.99792458 10^8

(%i11)  m[S]: M: 1.98855e30;
(%o11)  1.98855 10^30

(%i12)  m[M]: m: 3.301e21;
(%o12)  3.301 10^21

(%i13)  G: 6.67384e-11;
(%o13)  6.67384 10^-11
```

semi-major axis (große Halbachse)

```
(%i14)  a: 57.909175e9;
(%o14)  5.7909175 10^10

(%i15)  epsilon: 0.20563069;
(%o15)  0.20563069
```

velocities

```
(%i19)  v[min]: 38860;
        v[max]: 58980;
        v[av]:  47872.5;
        v[av_n]:(v[min]+v[max])/2.;
(%o16)  38860
(%o17)  58980
(%o18)  47872.5
(%o19)  48920
```

Delta_phi in arcsec

```
(%i20)  as_fac: 2*%pi/(360*3600);
(%o20)  %pi/648000
```

Delta_phi for 100 earth years

```
(%i21)  T_sideral: 87.969*24*3600;
(%o21)  7600521.6

(%i22)  T_earth: 365.256*24*3600;
(%o22)  3.15581184 10^7
```

semi latus rectum

```
(%i23)  alpha: a*(1-epsilon^2);
```
$$(\%o23)\quad 5.546054456369153\,10^{10}$$

perihelion and aphelion radius

```
(%i25)  r[p]: alpha/(1+epsilon);
        r[a]: alpha/(1-epsilon);
```
$$(\%o24)\quad 4.600127138741925\,10^{10}$$
$$(\%o25)\quad 6.981707861258075\,10^{10}$$

vN calculated:

```
(%i28)  vN2: m[S]*G*(2/r[p]-1/a);
        vN: sqrt(vN2);
        gamma:1/sqrt(1-vN2/c^2);
```
$$(\%o26)\quad 3.47821773883918\,10^{9}$$
$$(\%o27)\quad 58976.4168023048$$
$$(\%o28)\quad 1.000000019350196$$

3 Evaluation of delta_phi

```
(%i29)  numer: true;
```
$$(\%o29)\quad \text{true}$$

```
(%i30)  cos_delta_phi;
```
$$(\%o30)\quad -\frac{U\,\alpha}{G\,M\,\epsilon\,m}+\frac{5\,G^2\,M^2\left(\epsilon+1\right)^6}{16\,\alpha^2\,c^4\,\epsilon}+\frac{3\,G\,M\left(\epsilon+1\right)^4}{8\,\alpha\,c^2\,\epsilon}+\frac{\left(\epsilon+1\right)^2}{2\,\epsilon}-\frac{\epsilon}{2}-\frac{1}{2\,\epsilon}$$

```
(%i32)  ev(cos_delta_phi, [U=0]);
        cdp1: ev(cos_delta_phi-1, [U=0]);
```
$$(\%o31)\quad 1.000000102585811$$
$$(\%o32)\quad 1.025858114012124\,10^{-7}$$

```
(%i33)  Delta_phi: acos(cdp1)-%pi/2;
```
$$(\%o33)\quad -1.025858114012124\,10^{-7}$$

```
(%i34)  Delta_phi_ey: Delta_phi/as_fac*T_earth/T_sideral*100;
```
$$(\%o34)\quad -8.785776167398781$$

4 Evaluation of delta_phi with U

```
(%i35)  f: -42.98141712844573/Delta_phi_ey;
```
$$(\%o35)\quad 4.892159361848539$$

```
(%i36)  cdp2: f*cdp1;
```
$$(\%o36)\quad 5.018661376392697\,10^{-7}$$

```
(%i37)  Delta_phi: acos(cdp2)-%pi/2;
(%o37)  -5.018661375366662 10^-7

(%i38)  Delta_phi_ey: Delta_phi/as_fac*T_earth/T_sideral*100;
(%o38)  -42.98141711965845

(%i39)  cdp2U: ev(cos_delta_phi-1, [U=U2]);
(%o39)  1.025858114012124 10^-7 - 6.156563546624906 10^-31 U2

(%i40)  U1: solve(cdp2=cdp2U, U2);
rat: replaced 3.992803262380573E-7 by 13683/34269156532 = 3.992803262380569E
rat: replaced 6.156563546624906E-31 by 1/1624282755187690119545835487232 = 6
rat: replaced 3.992803262380569E-7 by 13683/34269156532 = 3.992803262380569E
rat: replaced 6.156563546624906E-31 by 1/1624282755187690119545835487232 = 6
rat: replaced 7.186127900026957E-41 by 1/139156999974387973036877151029961 32
rat: replaced 6.485441483941908E+23 by 648544148394190819033088/1 = 6.485441
(%o40)  [U2 = -648544148394190819033088]

(%i41)  printf(true, "~e", rhs(first(U1)));
(%o41)  false
-6.485441483941908E+23

(%i53)  ev(H_0); ev(H_r,[delta_phi=0],eval);
(%o52)  U - 3.782513239062031 10^30
(%o53)  -3.782513072433668 10^30
```