368(7): Investigation of Gyroscope Lift.

In the absence of a lifting force the lagrangian is:

$$\mathcal{L} = T - U \quad -(1)$$

where the rotational kinetic energy is:

$$T = \frac{1}{2} I_{12} \left(\dot{\phi}^2 \sin^2\theta + \dot{\theta}^2 \right) + \frac{1}{2} I_3 \left(\dot{\phi} \cos\theta + \dot{\psi} \right)^2 \quad -(2)$$

and the gravitational potential energy is:

$$U = mgh \cos\theta \quad -(3)$$

The Euler Lagrange equations of motion are:

$$\ddot{\theta} = \frac{\sin\theta}{I_{12}} \left(\dot{\phi}^2 \cos\theta \left(I_{12} - I_3 \right) - I_3 \dot{\phi} \dot{\psi} + mgh \right) \quad -(4)$$

$$\dot{\phi} = \frac{L_\phi - L_\psi \cos\theta}{I_{12} \sin^2\theta} \quad -(5)$$

and

$$\dot{\psi} = \frac{1}{I_3} \left(L_\psi - I_3 \dot{\phi} \cos\theta \right) \quad -(6)$$

which have been solved numerically by Horst Eckardt to give nutation and precession in a well defined way. This is the first complete solution of gyroscope motion since "Mecanique Analytique" (1811 and 1815) by Lagrange.

The simplest way to investigate lift in the gyroscope is to decrease the effective

) mass : $m \longrightarrow m - m_1$ — (7)

so eq. (4) becomes :

$$\ddot{\theta} = \frac{\sin\theta}{I_{12}} \left(\dot{\phi}^2 \cos\theta \left(I_{12} - I_3 \right) - I_3 \dot{\phi} \dot{\psi} + (m - m_1) g l \right) \quad - (8)$$

Eqs. (5) and (6) remain the same.

Therefore solve the equations (8) and (5) and (6) simultaneously to find the effect of m_1 on the nutation and precession of the gyroscope.

The gyroscope appears to be weightless when :

$$m = m_1 \quad - (9)$$

so under the weightless condition :

$$\ddot{\theta} = \frac{\sin\theta}{I_{12}} \left(\dot{\phi}^2 \cos\theta \left(I_{12} - I_3 \right) - I_3 \dot{\phi} \dot{\psi} \right) \quad - (10)$$

$$\dot{\phi} = \frac{L_\phi - L_\psi \cos\theta}{I_{12} \sin^2\theta} \quad - (11)$$

$$\dot{\psi} = \frac{1}{I_3} \left(L_\psi - I_3 \dot{\phi} \cos\theta \right) \quad - (12)$$

Solving eqs. (10) to (12) gives a well defined nutation and precession, the motion of a weightless gyroscope.