356(6): Equations for a Static Electric Field in the Cartesian Coordinate System.

Consider a static electric field along the Z axis:

$$\underline{E} = -\frac{e^2}{4\pi\epsilon_0 Z^2}\,\underline{k} \quad -(1)$$

$$= x(\underline{v}\cdot\underline{\nabla})\underline{v}$$

It follows that:

$$E_Z = x\left(v_X\frac{\partial}{\partial X} + v_Y\frac{\partial}{\partial Y} + v_Z\frac{\partial}{\partial Z}\right)v_Z = -\frac{e^2}{4\pi\epsilon_0 Z^2} \quad -(2)$$

$$E_Y = x\left(v_X\frac{\partial}{\partial X} + v_Y\frac{\partial}{\partial Y} + v_Z\frac{\partial}{\partial Z}\right)v_Y = 0 \quad -(3)$$

$$E_X = x\left(v_X\frac{\partial}{\partial X} + v_Y\frac{\partial}{\partial Y} + v_Z\frac{\partial}{\partial Z}\right)v_X = 0 \quad -(4)$$

In general there are three differential equations in three unknowns, v_X, v_Y and v_Z. If it is assumed that v_X, v_Y and v_Z are linearly independent so that:

$$\frac{\partial v_Z}{\partial Y} = \frac{\partial v_Z}{\partial X} = \frac{\partial v_Y}{\partial X} = \frac{\partial v_Y}{\partial Z} = \frac{\partial v_X}{\partial Y} = \frac{\partial v_X}{\partial Z} = 0$$

$$-(5)$$

then:

$$v_Z\frac{\partial v_Z}{\partial Z} = -\frac{e^2}{4\pi\epsilon_0 x Z^2} \quad -(6)$$

$$v_Y\frac{\partial v_Y}{\partial Y} = 0 \quad -(7)$$

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$$v_x \frac{\partial v_x}{\partial x} = 0 \quad -(8)$$

So

$$\frac{\partial v_y}{\partial y} = \frac{\partial v_x}{\partial x} = 0 \quad -(9)$$

and

$$v_z \frac{\partial v_z}{\partial z} = \frac{-e^2}{4\pi\epsilon_0 x z^2} \quad -(10)$$
