356(5): Particular Solution of Note 356(4)

If it is assumed that:

$$V_r = f(r), \quad V_\phi = f(\phi), \quad V_\theta = f(\theta) \quad -(1)$$

then:

$$\frac{-e^2}{4\pi\epsilon_0 r^2} = x V_r \left(\frac{2}{r} + \frac{\partial}{\partial r} \right) V_r \quad -(2)$$

$$\frac{V_\phi}{r \sin\theta} \frac{\partial V_\phi}{\partial \phi} = 0 \quad -(3)$$

$$\frac{V_\theta}{r} \left(1 + \frac{\cos\theta}{\sin\theta} \right) \frac{\partial V_\theta}{\partial \theta} = 0 \quad -(4)$$

So:

$$\frac{\partial V_\phi}{\partial \phi} = 0 \quad -(5)$$

$$\frac{\partial V_\theta}{\partial \theta} = 0 \quad -(6)$$

and

$$\frac{-e^2}{4\pi\epsilon_0 r^2} = \frac{2x}{r} V_r^2 + x V_r \frac{\partial V_r}{\partial r} \quad -(7)$$

Eq. (7) is a non linear partial differential equation for V_r.