356(2): Magnetic Field of a Circular Current Loop

In this case:

$$B_r = \frac{\mu_0 I a^2 \cos\theta}{2(a^2+r^2)^{3/2}} \left(1 + \frac{15 a^2 r^2 \sin^2\theta}{4(a^2+r^2)^2} + \dots \right) \quad -(1)$$

and

$$B_\theta = -\frac{\mu_0 I a^2 \sin\theta}{4(a^2+r^2)^{5/2}} \left(2a^2 - r^2 + \frac{15 a^2 r^2 \sin^2\theta (4a^2 - 3r^2)}{8(a^2+r^2)^2} + \dots \right) \quad -(2)$$

Far from the loop these equations simplify to:

$$B_r = \frac{\mu_0}{2\pi} \left(I \pi a^2 \right) \frac{\cos\theta}{r^3} \quad -(3)$$

and

$$B_\theta = \frac{\mu_0}{4\pi} \left(I \pi a^2 \right) \frac{\sin\theta}{r^3} \quad -(4)$$

Then the v_r and v_θ components of the induced spacetime velocity field are given by:

$$\left(\underline{B} \cdot \right) = \left(\frac{\rho_m}{\rho} \right) \left(\underline{\nabla} \times \underline{v} \right)_{\text{Cartesian}} \quad -(5)$$

and

$$\left(\underline{B} \cdot \right) = \left(\frac{\rho_m}{\rho} \right) \left(\underline{\nabla} \times \underline{v} \right)_{\text{plane polar}} \quad -(6)$$

From the theory of magnetostatics, the magnetic

2) field in ECE2 is defined by:

$$\underline{B} = \underline{\nabla} \times \underline{W} \qquad -(7)$$

and for components of the type (1) and (2):

$$W_\phi(r,\theta) = \frac{\mu_0 I a^2 r \sin\theta}{4(a^2+r^2)^{3/2}} \left(\underline{1} + \frac{15 a^2 r^2 \sin^2\theta}{8(a^2+r^2)^2} + \ldots \right) \qquad -(8)$$

If it is assumed that ρ_m / ρ is independent of distance then:

$$\underline{\nabla} \times \underline{W} = \frac{\rho_m}{\rho} \left(\underline{\nabla} \times \underline{V} \right)_{\text{spacetime}} \qquad -(9)$$

$$= \underline{\nabla} \times \left(\frac{\rho_m}{\rho} \underline{V} \right)_{\text{spacetime}}$$

and $\underline{W}(\text{loop}) = \frac{\rho_m}{\rho} \underline{V}(\text{spacetime}) \qquad -(10)$

and $\underline{V}$ can be worked out analytically:

$$V_\phi(r,\theta) = \frac{\rho}{\rho_m} W_\phi \qquad -(11)$$

For example we can assume:

$$\frac{\rho_m}{\rho} = \frac{m}{e} \qquad -(12)$$

where m and e are the mass and charge of an electron.
