```
(%i29) kill(all);
```

(%o0) *done*

1 Diff. Operators in Spherical coordinates

```
(%i1) grad_s(psi) := [diff(psi,r), 1/r*diff(psi,theta), 1/(r*sin(theta))*di
```

(%o1) $\mathrm{grad\_s}(\Psi):=[\mathrm{diff}(\Psi,r),\frac{1}{r}\mathrm{diff}(\Psi,\theta),\frac{1}{r\sin(\theta)}\mathrm{diff}(\Psi,\phi)]$

```
(%i2) div_s(a) := 1/r^2*diff(r^2*a[1],r) + 1/(r*sin(theta))*diff(sin(theta)
      + 1/(r*sin(theta))*diff(a[3],phi);
```

(%o2) $\mathrm{div\_s}(a):=\frac{1}{r^2}\mathrm{diff}\left(r^2 a_1,r\right)+\frac{1}{r\sin(\theta)}\mathrm{diff}(\sin(\theta)a_2,\theta)+\frac{1}{r\sin(\theta)}$
$\mathrm{diff}(a_3,\phi)$

```
(%i3) curl_s(a) := [1/(r*sin(theta))*(diff(sin(theta)*a[3],theta) - diff(a[
                    1/(r*sin(theta))*diff(a[1],phi) - 1/r*diff(r*a[3],r),
                    1/r*(diff(r*a[2],r) - diff(a[1],theta))];
```

(%o3) $\mathrm{curl\_s}(a):=[\frac{1}{r\sin(\theta)}\left(\mathrm{diff}(\sin(\theta)a_3,\theta)-\mathrm{diff}(a_2,\phi)\right),\frac{1}{r\sin(\theta)}$
$\mathrm{diff}(a_1,\phi)-\frac{1}{r}\mathrm{diff}(r\,a_3,r),\frac{1}{r}(\mathrm{diff}(r\,a_2,r)-\mathrm{diff}(a_1,\theta))]$

```
(%i4) Delta_s(psi) := 1/r^2*diff(r^2*diff(psi,r),r)
      + 1/(r^2*sin(theta))*diff(sin(theta)*diff(psi,theta),theta)
      + 1/(r^2*sin(theta)^2)*diff(psi,phi,2);
```

(%o4) $\mathrm{Delta\_s}(\Psi):=\frac{1}{r^2}\mathrm{diff}(r^2\,\mathrm{diff}(\Psi,r),r)+\frac{1}{r^2\sin(\theta)}$
$\mathrm{diff}(\sin(\theta)\mathrm{diff}(\Psi,\theta),\theta)+\frac{1}{r^2\sin(\theta)^2}\mathrm{diff}(\Psi,\phi,2)$

```
(%i5) a_nabla_s_b(a,b) :=[a[1]*diff(b[1],r) + a[2]/r*diff(b[1],theta) + a[3
                          a[1]*diff(b[2],r) + a[2]/r*diff(b[2],theta) + a[3
                          a[1]*diff(b[3],r) + a[2]/r*diff(b[3],theta) + a[3
```

(%o5) $\mathrm{a\_nabla\_s\_b}(a,b):=[a_1\,\mathrm{diff}(b_1,r)+\frac{a_2}{r}\mathrm{diff}(b_1,\theta)+\frac{a_3}{r\sin(\theta)}\mathrm{diff}(b_1,\phi)$
$,a_1\,\mathrm{diff}(b_2,r)+\frac{a_2}{r}\mathrm{diff}(b_2,\theta)+\frac{a_3}{r\sin(\theta)}\mathrm{diff}(b_2,\phi),a_1\,\mathrm{diff}(b_3,r)+\frac{a_2}{r}\mathrm{diff}(b_3,\theta)$
$+\frac{a_3}{r\sin(\theta)}\mathrm{diff}(b_3,\phi)]$

2 Diff. equations

2.1 Definitions

```
(%i6) E[r]: -e^2/(4*%pi*epsilon[0]*r^2);
```

(%o6) $-\frac{e^2}{4\,\pi\,\varepsilon_0\,r^2}$

```
(%i7) E[theta]: 0;
```

(%o7) 0

```
(%i8) E[phi]: 0;
```

(%o8) 0

```
(%i9) a: [v[r], v[theta], v[phi]];
```

(%o9) $[v_r, v_\theta, v_\phi]$

2.2 Full dependence

```
(%i10) depends(v,[r,theta,phi]);
```

(%o10) $[v(r,\theta,\phi)]$

```
(%i11) E[r] = x*a_nabla_s_b(a,[v[r],0,0]);
```

(%o11) $-\frac{e^2}{4\,\pi\,\varepsilon_0\,r^2}=[\left(\frac{v_\phi\left(\frac{d}{d\,\phi}v_r\right)}{r\,\sin(\theta)}+\frac{\left(\frac{d}{d\,\theta}v_r\right)v_\theta}{r}+v_r\left(\frac{d}{d\,r}v_r\right)\right)x,0,0]$

```
(%i12) E[theta] = x*a_nabla_s_b(a,[0,v[theta],0]);
```

(%o12) $0=[0,\left(\frac{v_\theta\left(\frac{d}{d\,\theta}v_\theta\right)}{r}+v_r\left(\frac{d}{d\,r}v_\theta\right)+\frac{v_\phi\left(\frac{d}{d\,\phi}v_\theta\right)}{r\,\sin(\theta)}\right)x,0]$

```
(%i13) E[phi] = x*a_nabla_s_b(a,[0,0,v[phi]]);
```

(%o13) $0=[0,0,\left(\frac{v_\phi\left(\frac{d}{d\,\phi}v_\phi\right)}{r\,\sin(\theta)}+\frac{\left(\frac{d}{d\,\theta}v_\phi\right)v_\theta}{r}+\left(\frac{d}{d\,r}v_\phi\right)v_r\right)x]$

2.3 Only r dependence

```
(%i14) depends(v,[r]);
```

(%o14) $[v(r)]$

```
(%i15) E[r] = x*a_nabla_s_b(a,[v[r],0,0]);
```

(%o15) $-\frac{e^2}{4\,\pi\,\varepsilon_0\,r^2}=[v_r\left(\frac{d}{d\,r}v_r\right)x,0,0]$

```
(%i16) E[theta] = x*a_nabla_s_b(a,[0,v[theta],0]);
```

(%o16) $0=\left[0,v_r\left(\frac{d}{d\,r}v_\theta\right)x,0\right]$

```
(%i17) E[phi] = x*a_nabla_s_b(a,[0,0,v[phi]]);
```

(%o17) $0=\left[0,0,\left(\frac{d}{d\,r}v_\phi\right)v_r\,x\right]$

2.4 Solution of r component

```
(%i18) E1: -e^2/(4*%pi*epsilon[0]*r^2)=v[r]*('diff(v[r],r,1))*x;
```

(%o18) $-\frac{e^2}{4\,\pi\,\varepsilon_0\,r^2}=v_r\left(\frac{d}{d\,r}v_r\right)x$

```
(%i19) E2: ode2(E1/v[r], v[r], r);
```

(%o19) $-\frac{2\,\pi\,\varepsilon_0\,v_r^2\,x}{e^2}=\%c-\frac{1}{r}$

```
(%i20) solve(E2, v[r]); radcan(%);
```

(%o20) $\left[v_r=-\frac{e\sqrt{\frac{1}{\varepsilon_0\,r\,x}-\frac{\%c}{\varepsilon_0\,x}}}{\sqrt{2}\,\sqrt{\pi}},v_r=\frac{e\sqrt{\frac{1}{\varepsilon_0\,r\,x}-\frac{\%c}{\varepsilon_0\,x}}}{\sqrt{2}\,\sqrt{\pi}}\right]$

(%o21) $\left[v_r=-\frac{e\sqrt{1-\%c\,r}}{\sqrt{2}\,\sqrt{\pi}\,\sqrt{\varepsilon_0}\,\sqrt{r}\,\sqrt{x}},v_r=\frac{e\sqrt{1-\%c\,r}}{\sqrt{2}\,\sqrt{\pi}\,\sqrt{\varepsilon_0}\,\sqrt{r}\,\sqrt{x}}\right]$

2.5 Check

```
(%i22) v[r]: -(e*sqrt(1/(epsilon[0]*r*x)-%c/(epsilon[0]*x)))/(sqrt(2)*sqrt(%
```

(%o22) $-\frac{e\sqrt{\frac{1}{\varepsilon_0\,r\,x}-\frac{\%c}{\varepsilon_0\,x}}}{\sqrt{2}\,\sqrt{\pi}}$

```
(%i23) ratsimp(-e^2/(4*%pi*epsilon[0]*r^2)=v[r]*('diff(v[r],r,1))*x);
```

(%o23) $-\frac{e^2}{4\,\pi\,\varepsilon_0\,r^2}=-\frac{e\left(\frac{d}{d\,r}\left(-\frac{e\sqrt{-\frac{\%c\,r-1}{\varepsilon_0\,r\,x}}}{\sqrt{2}\,\sqrt{\pi}}\right)\right)\sqrt{-\frac{\%c\,r-1}{\varepsilon_0\,r\,x}}\,x}{\sqrt{2}\,\sqrt{\pi}}$

```
(%i24) ev(%,diff);
```

(%o24) $-\frac{e^2}{4\,\pi\,\varepsilon_0\,r^2}=\frac{e^2\left(\frac{\%c\,r-1}{\varepsilon_0\,r^2\,x}-\frac{\%c}{\varepsilon_0\,r\,x}\right)x}{4\,\pi}$

```
(%i25) ratsimp(%);
```

(%o25) $-\frac{e^2}{4\,\pi\,\varepsilon_0\,r^2}=-\frac{e^2}{4\,\pi\,\varepsilon_0\,r^2}$

Solution o.k.