```
(%i1) kill(all);
(%o0) done
```

1 Diff. Operators in Spherical coordinates

```
(%i1) grad_s(psi) := [diff(psi,r), 1/r*diff(psi,theta), 1/(r*sin(theta))*di
```

$$\text{(\%o1)}\quad \text{grad\_s}(\Psi):=\left[\text{diff}(\Psi,r),\frac{1}{r}\,\text{diff}(\Psi,\theta),\frac{1}{r\,\sin(\theta)}\,\text{diff}(\Psi,\phi)\right]$$

```
(%i2) div_s(a) := 1/r^2*diff(r^2*a[1],r) + 1/(r*sin(theta))*diff(sin(theta)
      + 1/(r*sin(theta))*diff(a[3],phi);
```

$$\text{(\%o2)}\quad \text{div\_s}(a):=\frac{1}{r^2}\,\text{diff}\left(r^2\,a_1,r\right)+\frac{1}{r\,\sin(\theta)}\,\text{diff}(\sin(\theta)\,a_2,\theta)+\frac{1}{r\,\sin(\theta)}\,\text{diff}(a_3,\phi)$$

```
(%i3) curl_s(a) := [1/(r*sin(theta))*(diff(sin(theta)*a[3],theta) - diff(a[
                    1/(r*sin(theta))*diff(a[1],phi) - 1/r*diff(r*a[3],r),
                    1/r*(diff(r*a[2],r) - diff(a[1],theta))];
```

$$\text{(\%o3)}\quad \text{curl\_s}(a):=\left[\frac{1}{r\,\sin(\theta)}\left(\text{diff}(\sin(\theta)\,a_3,\theta)-\text{diff}(a_2,\phi)\right),\frac{1}{r\,\sin(\theta)}\,\text{diff}(a_1,\phi)-\frac{1}{r}\,\text{diff}(r\,a_3,r),\frac{1}{r}\left(\text{diff}(r\,a_2,r)-\text{diff}(a_1,\theta)\right)\right]$$

```
(%i4) Delta_s(psi) := 1/r^2*diff(r^2*diff(psi,r),r)
      + 1/(r^2*sin(theta))*diff(sin(theta)*diff(psi,theta),theta)
      + 1/(r^2*sin(theta)^2)*diff(psi,phi,2);
```

$$\text{(\%o4)}\quad \text{Delta\_s}(\Psi):=\frac{1}{r^2}\,\text{diff}(r^2\,\text{diff}(\Psi,r),r)+\frac{1}{r^2\,\sin(\theta)}\,\text{diff}(\sin(\theta)\,\text{diff}(\Psi,\theta),\theta)+\frac{1}{r^2\,\sin(\theta)^2}\,\text{diff}(\Psi,\phi,2)$$

```
(%i5) a_nabla_s_b(a,b) :=[a[1]*diff(b[1],r) + a[2]/r*diff(b[1],theta) + a[3
                          a[1]*diff(b[2],r) + a[2]/r*diff(b[2],theta) + a[3
                          a[1]*diff(b[3],r) + a[2]/r*diff(b[3],theta) + a[3
```

$$\text{(\%o5)}\quad \text{a\_nabla\_s\_b}(a,b):=\left[a_1\,\text{diff}(b_1,r)+\frac{a_2}{r}\,\text{diff}(b_1,\theta)+\frac{a_3}{r\,\sin(\theta)}\,\text{diff}(b_1,\phi),\ a_1\,\text{diff}(b_2,r)+\frac{a_2}{r}\,\text{diff}(b_2,\theta)+\frac{a_3}{r\,\sin(\theta)}\,\text{diff}(b_2,\phi),\ a_1\,\text{diff}(b_3,r)+\frac{a_2}{r}\,\text{diff}(b_3,\theta)+\frac{a_3}{r\,\sin(\theta)}\,\text{diff}(b_3,\phi)\right]$$

2 Diff. equation

2.1 Definitions and Diff.Eq.

```
(%i6) b: [v[r], v[theta], v[phi]];
```

(%o6) $[v_r, v_\theta, v_\phi]$

```
(%i7) depends([E,v],[r,theta,phi]);
```

(%o7) $[E(r,\theta,\phi), v(r,\theta,\phi)]$

```
(%i8) E1: E[phi] = x*a_nabla_s_b(b,[0,0,v[phi]]);
```

(%o8) $$E_\phi = \left[0, 0, \left(\frac{v_\phi\left(\frac{d}{d\,\phi}v_\phi\right)}{r\sin(\theta)} + \frac{\left(\frac{d}{d\,\theta}v_\phi\right)v_\theta}{r} + \left(\frac{d}{d\,r}v_\phi\right)v_r\right)x\right]$$

2.2 E[phi] of a current loop

```
(%i9) E[phi]: third(rhs(E1));
```

(%o9) $$\left(\frac{v_\phi\left(\frac{d}{d\,\phi}v_\phi\right)}{r\sin(\theta)} + \frac{\left(\frac{d}{d\,\theta}v_\phi\right)v_\theta}{r} + \left(\frac{d}{d\,r}v_\phi\right)v_r\right)x$$

```
(%i10) v[r]: v[theta]: 0;
```

(%o10) 0

```
(%i11) v[phi]: mu[0]*I*a^2*r*sin(theta)/(4*(a^2+r^2)^(3/2))*
       (1 + 15*a^2*r^2*sin(theta)^2/(8*(a^2+r^2)^2));
```

(%o11) $$\frac{\mu_0\, a^2\, r\sin(\theta)\left(\frac{15\,a^2\,r^2\sin(\theta)^2}{8\,(r^2+a^2)^2}+1\right)I}{4\,(r^2+a^2)^{3/2}}$$

```
(%i12) E1: ev(E[phi],diff);
```

(%o12) 0

3 Graphics of A[phi], v[phi]

```
(%i13) v1: ev(v[phi], [a=1, I=1, mu[0]=1,r=0.5]);
       v2: ev(v[phi], [a=1, I=1, mu[0]=1,r=1]);
       v3: ev(v[phi], [a=1, I=1, mu[0]=1,r=1.5]);
       v4: ev(v[phi], [a=1, I=1, mu[0]=1,r=2]);
```

$$(\%o13)\quad 0.089442719099992\,\sin(\theta)\left(0.3\,\sin(\theta)^2+1\right)$$

$$(\%o14)\quad \frac{\sin(\theta)\left(\frac{15\,\sin(\theta)^2}{32}+1\right)}{2^{7/2}}$$

$$(\%o15)\quad 0.064003868795219\,\sin(\theta)\left(0.39940828402367\,\sin(\theta)^2+1\right)$$

$$(\%o16)\quad \frac{\sin(\theta)\left(\frac{3\,\sin(\theta)^2}{10}+1\right)}{2\,5^{3/2}}$$

```
(%i18) wxplot2d([v1,v2,v3,v4], [theta,0,%pi], [ylabel, "v[phi]"],
       [legend, "r=0.5", "r=1.0", "r=1.5", "r=2.0"])$
```

(%t18)